Arithmetic and Geometric Sequences

A **sequence** is a set of numbers in a particular order or pattern.

- Example: Fibonacci Sequence: 0, 1, 1, 2, 3, 5, 8, 13, …

A **Term** is each number in a sequence.

- The first term is denoted by a_1 (Read as: a subscript 1)
- The second term is denoted by a_2, third term is denoted by a_3, and so on.
- All term subscripts are made of natural numbers

You try: What is a_5 of the Fibonacci Sequence?

A **Finite Sequence** contains a limited number of terms.

- Example: {-2, 0, 2, 4, 6}

An **Infinite Sequence** continues without end.

- Example: Fibonacci Sequence

Example #1:
{2, 4, 8, 16, 32}

Example #2:
2, 4, 8, 16, 32, …

The domain of a finite sequence is the subscripts of each term.
The range of a finite sequence is the values of the sequence.

Sections 12.1/12.3/12.4

Arithmetic Sequence

- Each term is determined by adding a constant value to the previous term.
- The added constant value is called the common difference.
    - Example: {3, 6, 9, 12, 15}

Geometric Sequence

- Each term is determined by multiplying a nonzero constant by the previous term.
- The multiplied nonzero constant is called the common ratio.

Sections 12.1/12.3/12.4

Determine whether each sequence is arithmetic or geometric.

(1) 2, 6, 18, 54, …

Find the common difference or ratio.	2, 6, 18, 54 x3
Difference = arithmetic Ratio= geometric	

(2) 5, -6, -17, -28, …

Find the common difference or ratio.	
Difference = arithmetic Ratio= geometric	

(3) -4, 12, 28, 42, …

Find the common difference or ratio.	
Difference = arithmetic Ratio= geometric	

Types of Formulas for Sequences

Explicit Formula

- Defines the *n*th term of a sequence as a function of *n.*

Arithmetic Sequence – Linear

- Formula: $\boldsymbol{a_n = a_1 + (n - 1)d}$

Geometric Sequence – Exponential

- Formula: $a_n = a_1 \left(r^{n-1}\right)$

I Try:

Write the explicit formula.

(1) 2, 6, 18, 54, …

Identify as arithmetic or geometric	2, 6, 18, 54, … Geometric
Find common difference or ratio	Geometric->ratio x3
Plug $a_1 and\ d$ into equation. Arithmetic: $\boldsymbol{a_n = a_1 + (n - 1)d}$ Geometric $\boldsymbol{a_n = a_1\ (r^{n-1})}$	$a_n = a_1\ (r^{n-1})$ $a_n = 2(3^{n-1})$

Sections 12.1/12.3/12.4

We Try:

5, -6, -17, -28, …

Identify as arithmetic or geometric	5, -6, -17, -28, …
Find common difference or ratio	
Plug $a_1 and\ d$ into equation Arithmetic: $\boldsymbol{a_n = a_1 + (n - 1)d}$ Geometric $\boldsymbol{a_n = a_1\ (r^{n-1})}$	

You Try with your partner in your notebook:
Even Talk, Odd Write

Write the explicit formula.

-4, 12, 28, 42, …

Identify as arithmetic or geometric	5, -6, -17, -28, …
Find common difference or ratio	
Plug $a_1 and\ d$ into equation Arithmetic: $\boldsymbol{a_n = a_1 + (n - 1)d}$ Geometric $\boldsymbol{a_n = a_1\ (r^{n-1})}$	

Recursive Formula

- A rule in which one or more previous terms are used to generate the next term.
    - Example: Fibonacci Sequence

0, 1, 1, 2, 3, 5, 8, 13,

I Try:
Find the first 5 terms in the sequence.

$a_n = 2a_{n-1} + a_{n-2}$ where $a_1 = 2$ and $a_2 = 4$

You try: **Find the first 3 terms of the sequence:** $a_n = n^2 - 2n$

I Try:

Find the indicated terms in this arithmetic sequence:

11, __, __, __, -17

Find the common difference $\boldsymbol{a_n = a_1 + (n-1)d}$	$a_1 = 11, \ a_5 = -17$ $a_5 = a_1 + (a_5 - 1)d$ $-17 = 11 + (5-1)d$ $-17 = 11 + 4d$ $-28 = 4d$ $-7 = d$
Use the difference to find the missing terms	11, 4, -3 ,-10, -17

We Try:

Geometric Sequence: 4, ___, 16, …

Find the common ratio $\boldsymbol{a_n = a_1\ (r^{n-1})}$	$a_1 = 4 \quad a_3 = 16$
Use the ratio to find the missing term.	

You try with your partner:

Even Write, Odd Talk

Find the missing terms

Arithmetic Sequence: ½ , ___ , ____ , 2

I Try:

Find the 10th term of the geometric sequence that has a_5 = 96 and a_7 = 384.

Plug terms into formulas. $a_n = a_1 (r^{n-1})$	$a_5 = a_1(r^{5-1})$ $\quad 96 = a_1(r^4)$ $a_7 = a_1(r^{7-1})$ $\quad 384 = a_1(r^6)$
Solve for ***the common ratio***	$96 = a_1(r^4)$ $\quad a_1 = \frac{96}{r^4}$ $384 = a_1(r^6)$ $\quad a_1 = \frac{384}{r^6}$ $\frac{96}{r^4} = \frac{384}{r^6}$ $96r^6 = 384r^4$ $\frac{r^6}{r^4} = \frac{384}{96}$ $r^2 = 4$ $r = 2$
Use ratio to solve for a_1	$96 = a_1(r^4)$ $96 = a_1(2^4)$ $96 = a_1(16)$ $6 = a_1$
Use a_1 and r to find the missing term.	$a_n = a_1 (r^{n-1})$ $a_{10} = 6(2^{10-1})$ $a_{10} = 6(2^9)$ $a_{10} = 6(512)$ $a_{10} = 3072$

We Try:
Find the missing terms.
_____ ,-20 , _____, -2

Plug terms into formulas. $a_n = a_1 + (n-1)d$	
Solve for $common\ difference$	
Use difference to solve for a_1	
Use a_1 **and** d **to find the missing term.**	

You Try with your partner:
Even Write, Odd Talk

Find the 6th term of the arithmetic sequence with
$a_9 = 120 \; and \; a_{14} = 195$

Plug terms into formulas. $\boldsymbol{a_n = a_1 + (n-1)d}$	
Solve for $\boldsymbol{common\ difference}$	
Use difference to solve for $\boldsymbol{a_1}$	
Use $\boldsymbol{a_1}$ **and** $\boldsymbol{d}$ **to find the missing term.**	