<u>Introduction to Series and Summation Notation</u>

A **<u>series</u>** is the indicated sum of the terms of a sequence.

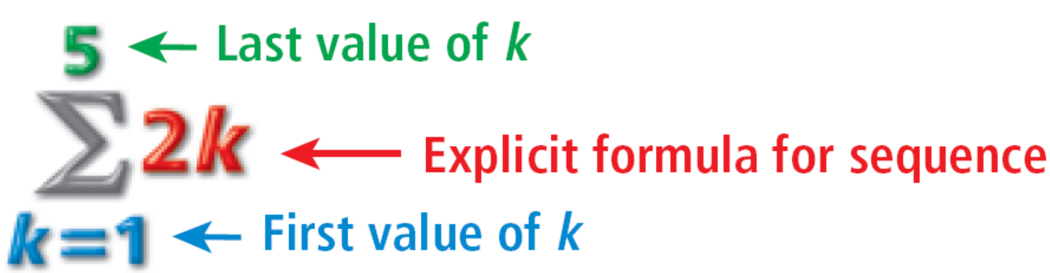
Sequence	1, 2, 3, 4	2, 4, 6, 8, …	$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}$
Series	$1 + 2 + 3 + 4$	$2 + 4 + 6 + 8 + \cdots$	$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$

A **<u>partial sum</u>**, indicated by S_n, is the sum of a specified number of terms of a sequence.

<u>Example #1:</u>

Find S_5 for the even numbers sequence: 2, 4, 6, 8, …

A series can also be represented by using **<u>summation notation</u>**, which uses the Greek letter Σ (capital *sigma*) to denote the sum of a sequence defined by a rule.



Write the series in summation notation:

I do:

1) 3 +6 +9 +12

Solve the formula	3+6+9+12 = 3k
Find the last value of k (The number of terms)	4 (There are 4 terms)
Find the first value of k.	K=1
Plug into sigma notation.	

We Try:

2) 4 + 9 +14 +19 +24 +29

Solve the formula.	
Find the last value of k (The number of terms).	
Find the first value of k.	K=1
Plug into sigma notation.	

We Try:

3) 8 +16+ 32 + 64 + 128

Solve the formula.	2^{K+2}
Find the last value of k (The number of terms).	
Find the first value of k.	K=1
Plug into sigma notation.	

You Try with your partner. In your notebooks.
Odd Talk, Even Write.

4) -1 + 2 - 4 + 8

Solve the formula.	$-1^{k}(2^{k-1})$
Find the last value of k (The number of terms).	
Find the first value of k.	K=1
Plug into sigma notation.	

For sequences using alternating signs, use $(-1)^{k+1}$ if a_1 is positive and use $(-1)^{k}$ if a_1 is negative.

I Try:

Example #2: **Expand the series and evaluate.**

$$\sum_{k=1}^{6}\left(k^2 - 10\right)$$

K=1	K=2	K=3	K=4	K=5	K=6
$(1^2 - 10)$	$(2^2 - 10)$	$(3^2 - 10)$	$(4^2 - 10)$	$(5^2 - 10)$	$(6^2 - 10)$
-9	-6	-1	6	15	26

-9-6-1+6+15+26=31

We Try:

Example #3: **Expand and evaluate.**

$$\sum_{k=1}^{5} -5(2)^{k-1}$$

K=1	K=2	K=3	K=4	K=5

You try with your partner in your notebooks:
Odd Write, Even Talk.

$$\sum_{k=1}^{7} 3$$

In a *constant series,* each term has the same value.

Arithmetic and Geometric Series

Arithmetic Series

- The sum of an arithmetic sequence.

$$S_n = n\left(\frac{a_1 + a_n}{2}\right)$$

- n is the number of terms, a_1 is the 1st term, and a_n is the last term.

I Try:

Example #1:

Find the sum of 2 + 4 + 6 + 8 + 10

Find first (a_1) and last (a_n) term.	$(a_1) = 2$ $(a_n) = 10$ n=5
Plug into equation $S_n = n\frac{(a_1+a_2)}{2}$	$S_n = 5\frac{(2+10)}{2}$
Simplify	$S_n = 5\left(\frac{12}{2}\right) = \frac{60}{2} = 30$

We Try:

Find S_{10} for 13 + 2 + (–9) + (–20) + ...

Find first (a_1) and last (a_n) term.	
Plug into equation $S_n = n\frac{(a_1+a_2)}{2}$	
Simplify	

You try with your partner
Odd Talk, Even Write:

$$\sum_{k=1}^{15}(5+2k)$$

Geometric Series – The sum of a geometric sequence.

$$S_n = a_1\left(\frac{1 - r^n}{1 - r}\right),\ r \neq 1$$

- n is the number of terms, a_1 is the 1st term, and a_n is the last term.

I Try:

Find $\boldsymbol{S_8}$ for **1 + 2 + 4 + 8 + 16 + ...**

Find first (a_1) and last (a_n) term.	$(a_1) = 1$ n=8 r=2 $(Use\ a_n = a_1 r^{n-1}\ to\ find\ a_8)$ $(a_8) = (a_1)r^{8-1}$ $(a_8) = (1)2^7$ $a_8 = 2^7$ $a_8 = 128$
Plug into equation $S_n = a_1(\frac{1-r^n}{1-r})$	$S_n = 1(\frac{1-2^8}{1-2})$
Simplify	$S_n = \frac{(1-256)}{-1} = 255$

We Try:

Example #4:

$$\sum_{k=1}^{6}\left(\frac{1}{2}\right)^{k-1}$$

Find first (a_1) and last (a_n) term.	
Plug into equation $S_n = a_1(\frac{1-r^n}{1-r})$	
Simplify	

Section 12.2/12.3/12.4

<u>You try with your partner:</u>
<u>Odd Write, Even Talk</u>

Find S_5 for the sequence: 32, 8, 2, .5, …

Find first (a_1) and last (a_n) term.	
Plug into equation $S_n = a_1(\frac{1-r^n}{1-r})$	
Simplify	