Ellipses, Hyperbolas, and Parabolas
10.3,10.4,10.5

An ***Ellipse*** is the set of points in a plane such that the sum of the distances from any point P on the ellipse to two fixed points called the Foci, is constant.

The ***major axis*** is the longer axis.
The ***minor axis*** is the shorter axis.
The ***vertices*** are the endpoints of the major axis.
The ***co-verticies*** are the endpoints of the minor axis.

Here are some examples of Ellipses.

$$\frac{x^2}{4}+\frac{y^2}{9}=1$$

$$\frac{(x-2)^2}{4} + \frac{(y+1)^2}{9} = 1$$

Standard Form for the Equation of an Ellipse — Center at (h, k)

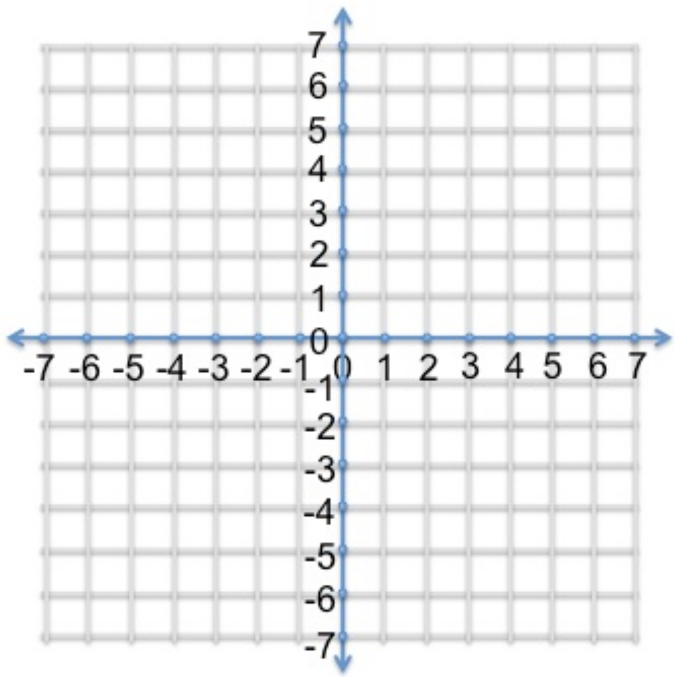
MAJOR AXIS	HORIZONTAL	VERTICAL
Equation	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$
Vertices	$(h+a, k), (h-a, k)$	$(h, k+a), (h, k-a)$
Foci	$(h+c, k), (h-c, k)$	$(h, k+c), (h, k-c)$
Co-vertices	$(h, k+b), (h, k-b)$	$(h+b, k), (h-b, k)$

Pg 738

Graph the Ellipse

Identify the Domain and Range

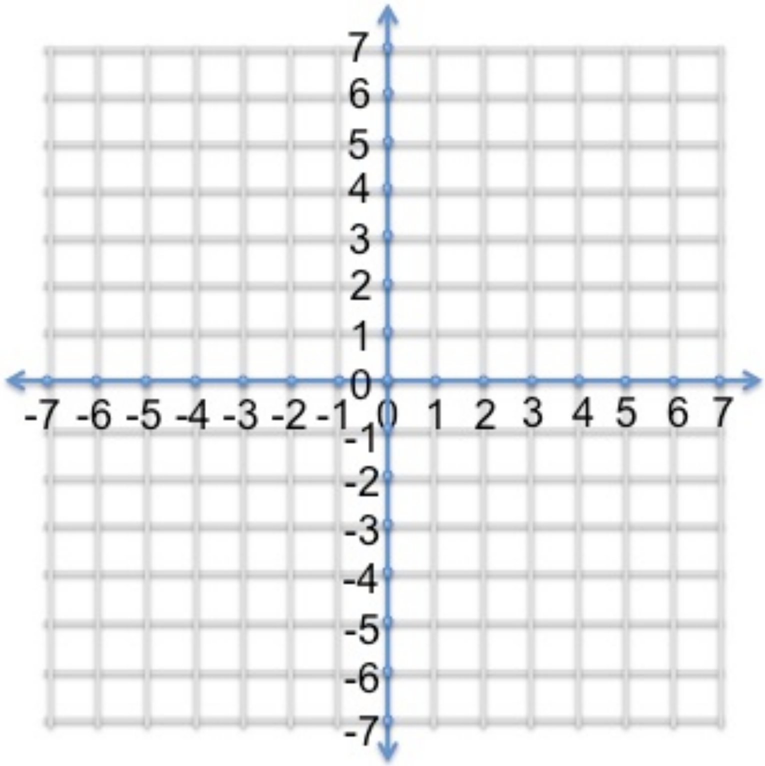
$$\frac{x^2}{64} + \frac{y^2}{25} = 1$$



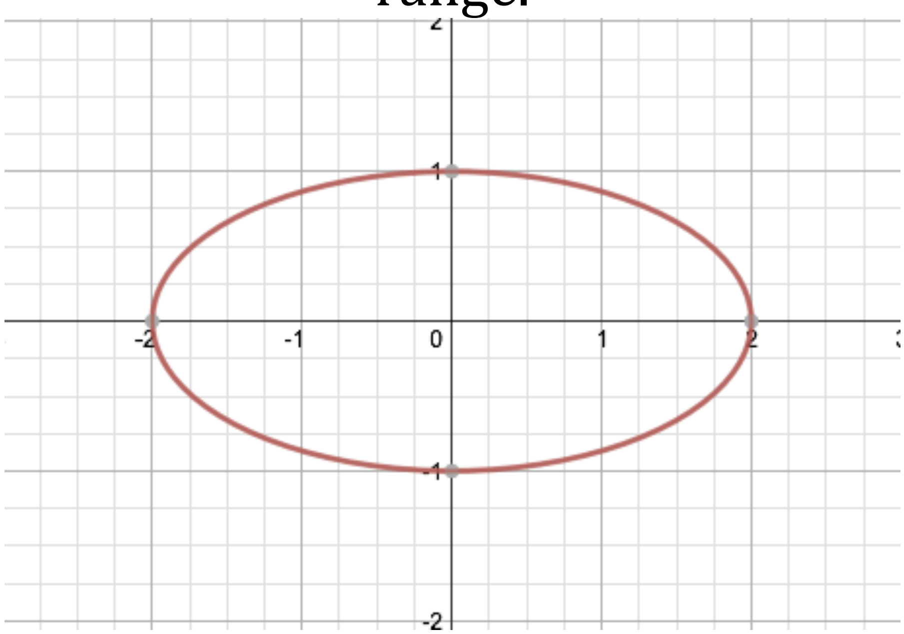
Graph the Ellipse

Identify the Domain and Range

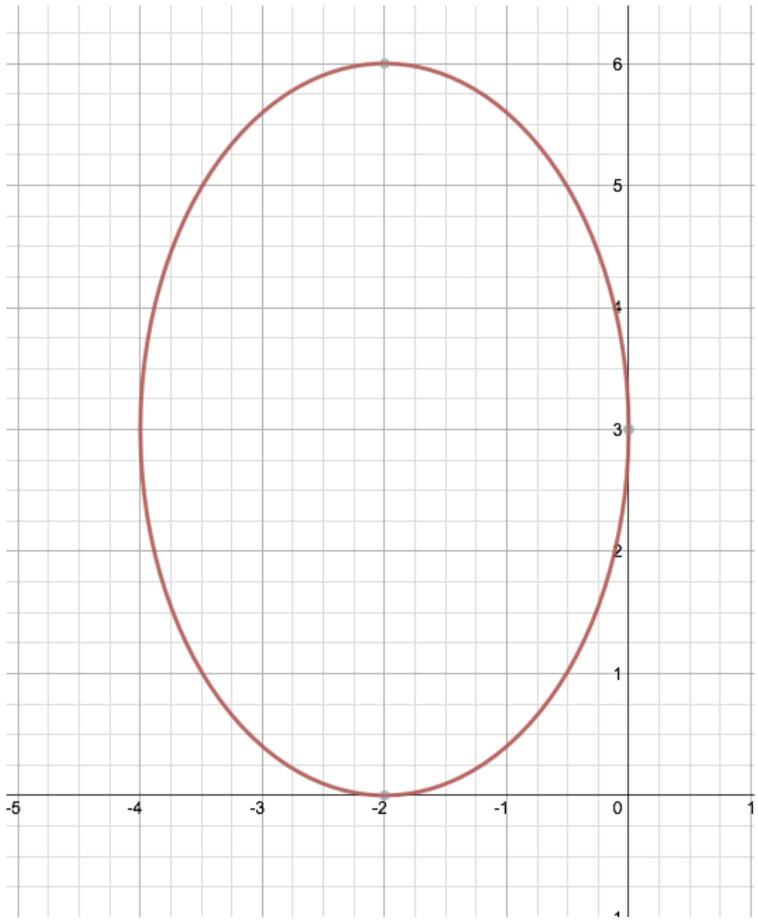
$$\frac{(x-1)^2}{4}+\frac{(y+2)^2}{25}=1$$



Write the equation and state the domain and range.

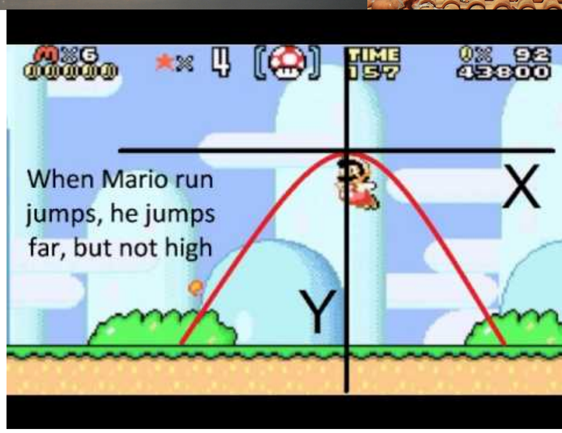


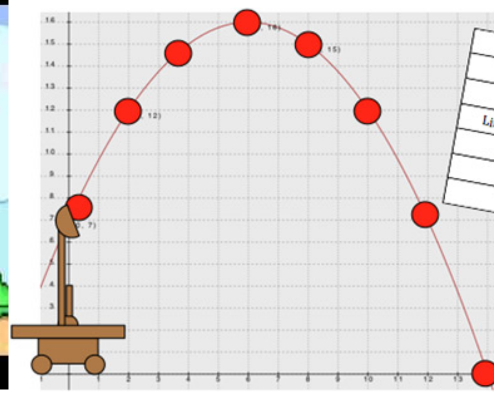
Write the equation and state the domain and range

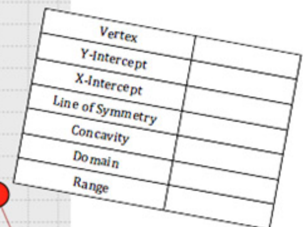


Parabolas

10.5







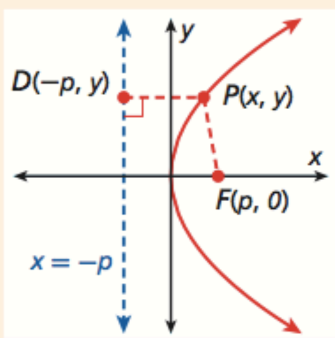
Vertex	
Y-Intercept	
X-Intercept	
Line of Symmetry	
Concavity	
Domain	
Range	

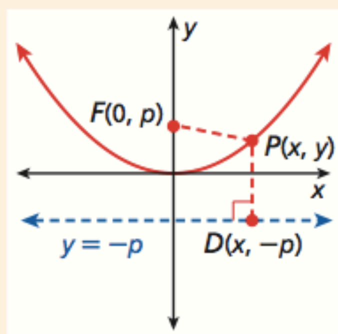
parabola is the set of all points P in a plane that are equal distance from both a fixed point,(focus) and a fixed line, the **directrix.**

he vertex is in the middle of the line from the directrix to focus.

tandard Form for the Equation of a Parabola — **Vertex at (0, 0)**

AXIS OF SYMMETRY	HORIZONTAL $y = 0$	VERTICAL $x = 0$
Equation	$x = \frac{1}{4p}y^2$	$y = \frac{1}{4p}x^2$
Direction	Opens right if $p > 0$ Opens left if $p < 0$	Opens upward if $p > 0$ Opens downward if $p < 0$
Focus	$(p, 0)$	$(0, p)$
Directrix	$x = -p$	$y = -p$
Graph	see graphs below	see graphs below





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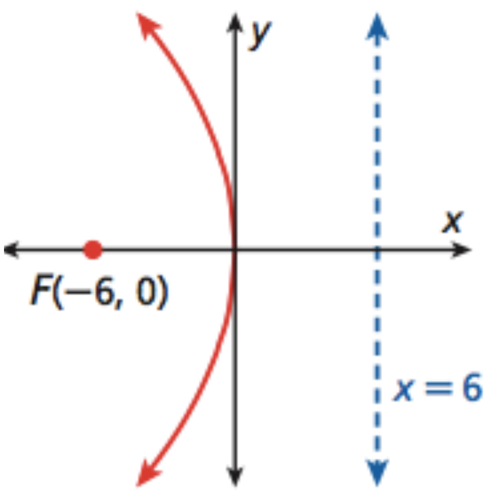
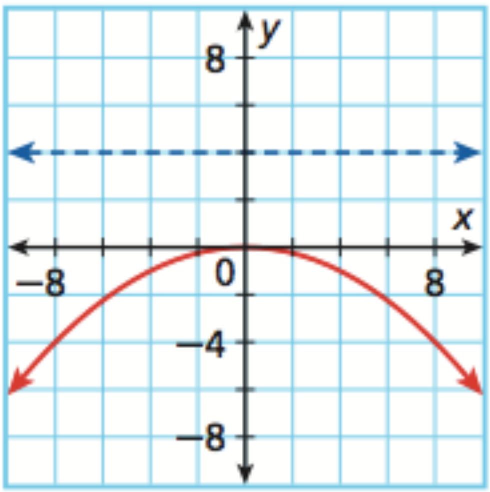
xample 1
try.
Vrite the equation in standard form for the parabola.



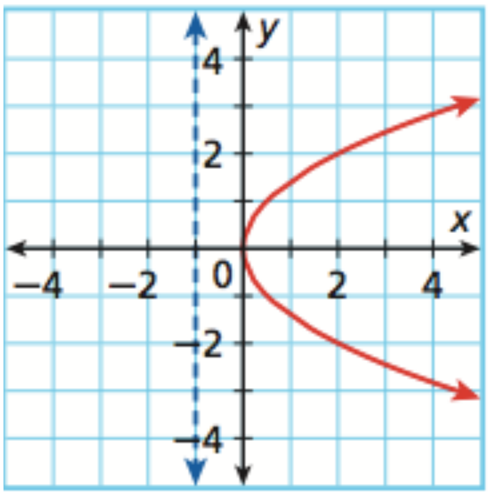
Axis of Symmetry	y = 0 Horizontal $x = \frac{1}{4p}y^2$
Focus value, p	Vertex is at (0,0) Directrix is at x=6 x = -p p =-6
Plug in “p” into equation	$x = \frac{1}{4(-6)}y^2$ $x = \frac{1}{-24}y^2$

Ve Try:
Vrite the equation in standard form.

5. 

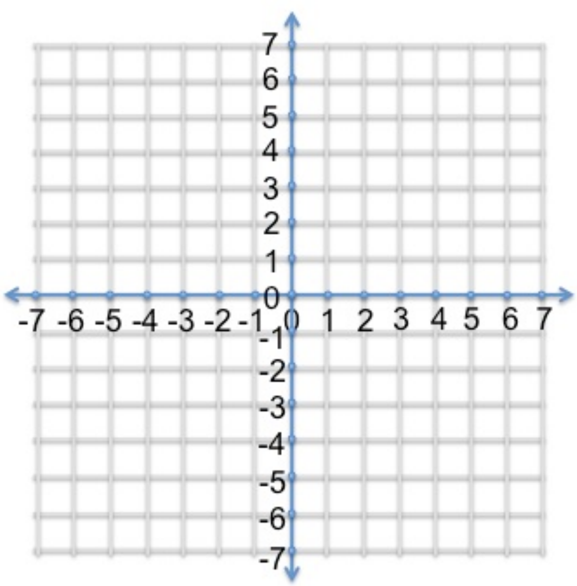
Axis of Symmetry	
Solve for p	Vertex is at Directrix is at p =
Plug in “p” into equation	

Ve Try:
Vrite the equation in standard form.



Axis of Symmetry	
Solve for p	Vertex is at Directrix is at p =
Plug in "p" into equation	

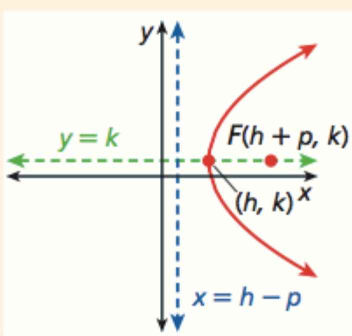
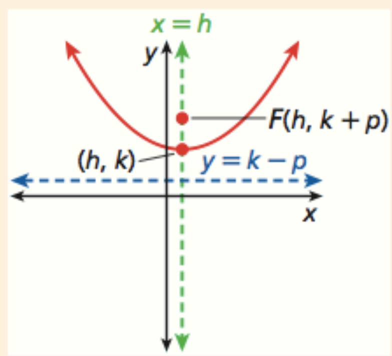
ou Try in your notebooks
)dd talk, Even write:
Vrite the equation in standard form.



Axis of Symmetry	
Solve for p	Vertex is at Directrix is at p =
Plug in “p” into equation	

)esmos

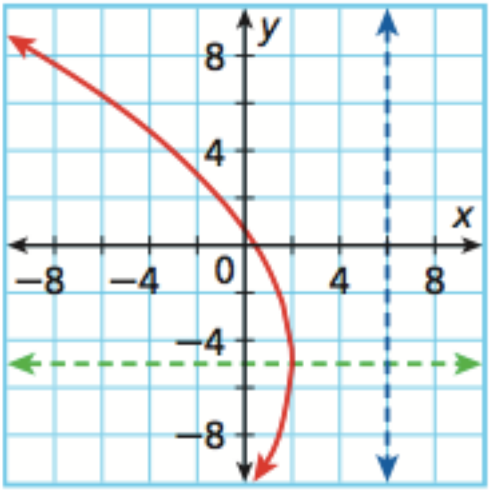
tandard Form for the Equation of a Parabola — Vertex at (h, k)

AXIS OF SYMMETRY	HORIZONTAL $y = k$	VERTICAL $x = h$
Equation	$x - h = \frac{1}{4p}(y - k)^2$	$y - k = \frac{1}{4p}(x - h)^2$
Direction	Opens right if $p > 0$ Opens left if $p < 0$	Opens upward if $p > 0$ Opens downward if $p < 0$
Focus	$(h + p, k)$	$(h, k + p)$
Directrix	$x = h - p$	$y = k - p$
Graph		

'g.753

Try.

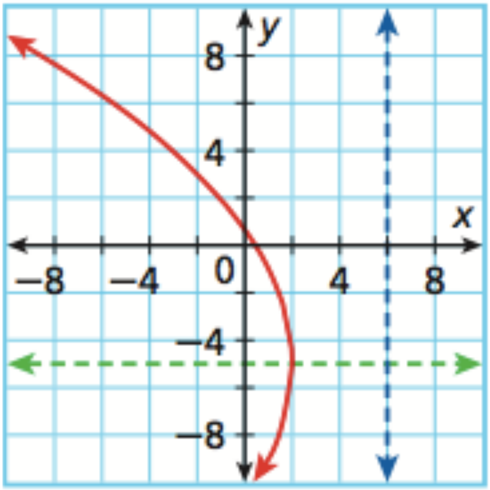
Vrite the equation in standard form.



Axis of Symmetry	Y=-5 Horizontal $x - h = \frac{1}{4p}(y - k)^2$
Solve for h,k,p	Vertex is at (2,-5) h=2, k=-5 Directrix is at x=6 x=h-p 6=2-p 4=-p -4=p
Plug in "h,k,p" into equation	$x - h = \frac{1}{4p}(y - k)^2$ $x - 2 = \frac{1}{4(-4)}(y - (-5))^2$ $x - 2 = \frac{1}{-16}(y + 5)^2$

We Try:

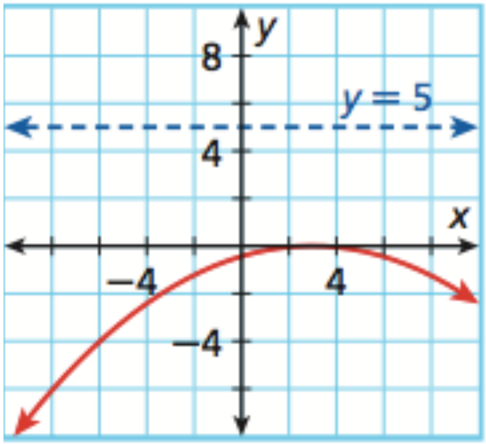
Vrite the equation in standard form.



Axis of Symmetry	
Solve for h,k,p	Vertex is at h= k= Directrix is at
Plug in "h,k,p" into equation	

'ou Try:

ven talk, Odd write:

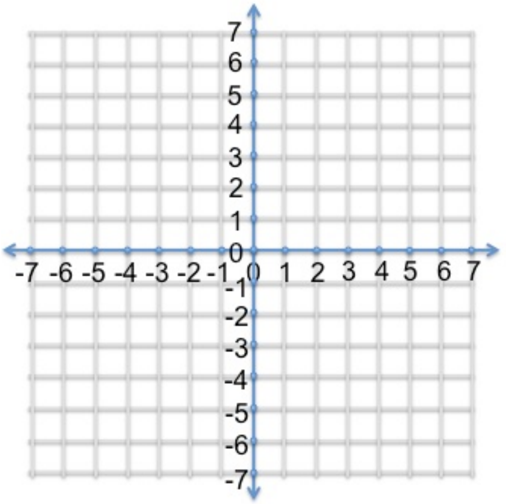


Axis of Symmetry	
Solve for h,k,p	Vertex is at h= k= Directrix is at
Plug in “h,k,p” into equation	

xample 2
ind the vertex, value of p, axis of symmetry, focus, and directrix, and then graph.

$$y = \frac{1}{32}(x+2)^2$$

Identify axis of symmetry	Vertical $y - k = \frac{1}{4p}(x-h)^2$
Vertex (h,k)	h=-2 k=0 (-2,0)
p	4p=32 p=8
focus	(h,k+p) (-2,0+8) (-2,8)
directrix	y=k-p y=0-8 y=-8

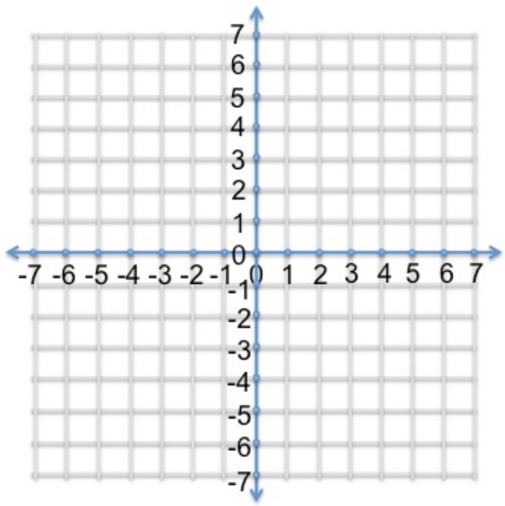


Ve Try:

ind the vertex, value of p, axis of symmetry, focus, and directrix, and then graph.

$$x = \frac{1}{8}(y-1)^2$$

Identify axis of symmetry	
Vertex (h,k)	h= k=
p	4p= p=
focus	
directrix	


7
6
5
4
3
2
1
0
-7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7
-1
-2
-3
-4
-5
-6
-7

ou Try in your notebooks with your partner:
ven Write, Odd Talk
ind the vertex, value of p, axis of symmetry, focus, and directrix, and then graph.

$$x - 2 = \frac{1}{2}\,(y + 1)^2$$

Identify axis of symmetry	
Vertex (h,k)	h= k=
p	4p= p=
focus	
directrix	

xample 3

Try:

Vrite the equation in standard form. Find the domain and range.

'ertex (-7,-3), Focus $(2, -3)$

Axis of Symmetry	Horizontal $x - h = \frac{1}{4p}(y - k)^2$
Find h,k,p	H=-7,k=-3 Focus (h+p,k)∂ $-7 + p = 2$ p=9
Plug h,k,p into equation	$x - h = \frac{1}{4p}(y - k)^2$ $x - (-7) = \frac{1}{4(9)}(y - (-3))^2$ $x + 7 = \frac{1}{36}(y + 3)^2$
Find the Domain and Range	$x \geq -7$ $\mathrm{y}: \mathrm{R}$

Try:

Vrite the equation in standard form. Find the domain and range.

ocus (4,-5), Directrix $x = 12$

Axis of Symmetry	Directrix $x = 12$ Horizontal $x - h = \frac{1}{4p}(y - k)^2$
Find h,k,p	Vertex is halfway between the focus and directrix. Vertex (8,-5) h=8 k=-5 Focus (h+p,k) 4=8+p -4=p
Plug h,k,p into equation	$x - h = \frac{1}{4p}(y - k)^2$ $x - 8 = \frac{1}{4(-4)}(y - (-5))^2$ $x - 8 = \frac{1}{-16}(y + 5)^2$

Find the Domain and Range	$x \leq 8$ y: R

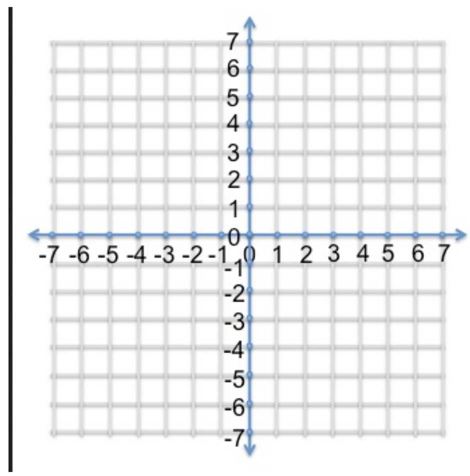
”

Ve Try:

Vrite the equation in standard form.

)dd Write Even Talk

'ertex (0,0), Focus $(\frac{1}{2}, 0)$

Axis of Symmetry	
Find p	Focus (p,0) p=
Plug p into equation	

Closure:

Discuss these questions with your partner.

How can we find the vertex given the focus and directrix?

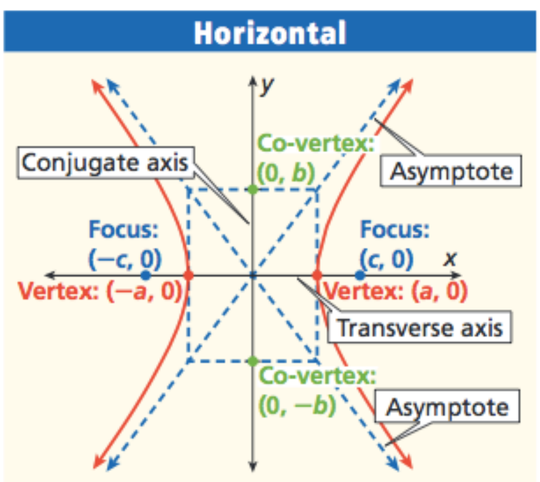
How can we tell if a parabola is vertical or horizontal?

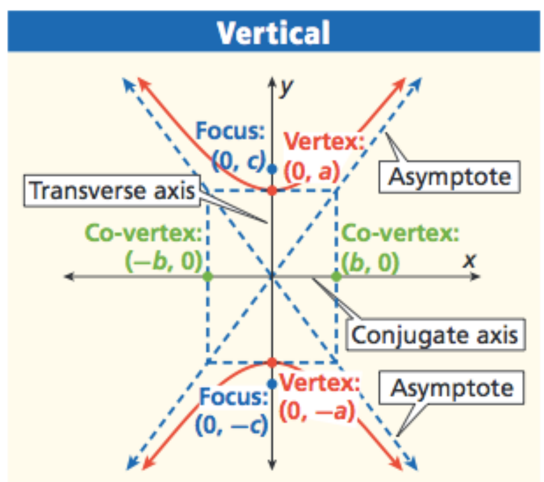
What are the steps needed to graph a parabola given the equation?

Hyperbolas

10.4


- A ***hyperbola*** is the set of points P in a plane such that the difference of the distances from P to the foci is constant.
- A hyperbola is made up of two **branches**.
- The **transverse axis** of symmetry contains the *vertices and foci.*
- The **conjugate axis** of symmetry separates the two branches of the hyperbola and contains the *co-vertices*.





Standard Form for the Equation of a Hyperbola Center at (0, 0)

TRANSVERSE AXIS	HORIZONTAL	VERTICAL
Equation	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$
Vertices	$(a, 0), (-a, 0)$	$(0, a), (0, -a)$
Foci	$(c, 0), (-c, 0)$	$(0, c), (0, -c)$
Co-vertices	$(0, b), (0, -b)$	$(b, 0), (-b, 0)$
Asymptotes	$y = \pm\frac{b}{a}x$	$y = \pm\frac{a}{b}x$

Pg. 745

Question!

For ellipses, are $\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$ and $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ the same?

What about for hyperbolas?

Are $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ and $\frac{x^2}{b^2} - \frac{y^2}{a^2} = 1$ the same?

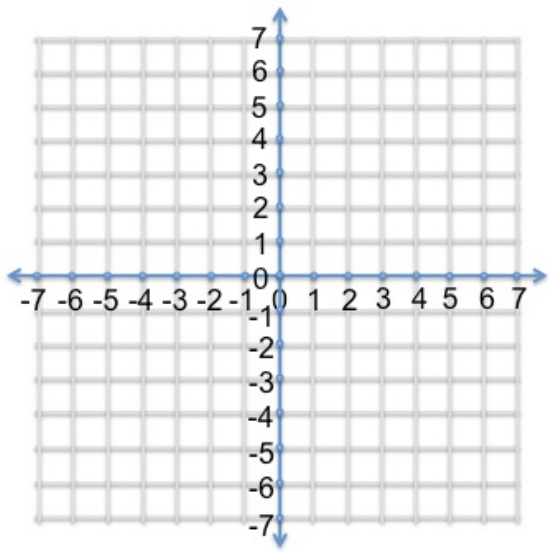
What do you remember about the "a" for ellipses

I do.

Find the vertices, co-vertices, and asymptotes of each hyperbola, and then graph.

$$\frac{y^2}{4} - \frac{x^2}{9} = 1$$

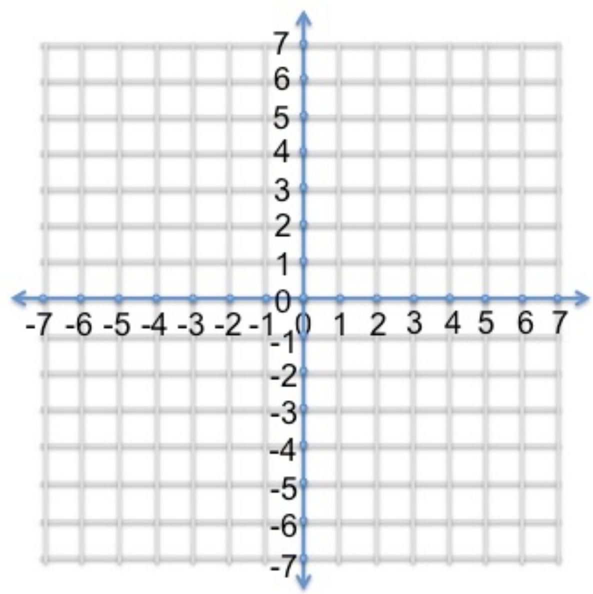
Identify form of the equation	Vertical
Identify "a" and "b"	$a^2 = 4, \quad a = 2$ $b^2 = 9, \quad b = 3$
Identify the Center, vertices, and co-vertices	Center: (0,0) Vertices: (0,2),(0,-2) Co-Vertices:(3,0),(-3,0)
Identify the Asymptotes: Vertical $y = \pm\frac{a}{b}x$ Horizontal $y = \pm\frac{b}{a}x$	$y = \pm\frac{2}{3}x$



We do.
Find the vertices, co-vertices, and asymptotes of each hyperbola, and then graph.

$$\frac{x^2}{25} - \frac{y^2}{1} = 1$$

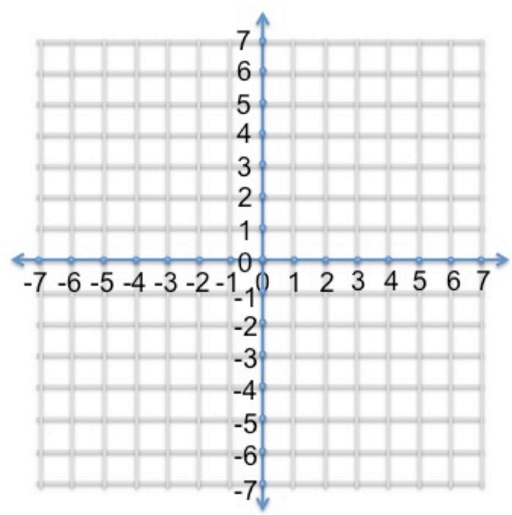
Identify form of the equation	
Identify the Center, vertices, and co-vertices	Center: Vertices: Co-Vertices:
Identify a and b	a= b=
Identify the Asymptotes	



We do.
Find the vertices, co-vertices, and asymptotes of each hyperbola, and then graph.

$$\frac{y^2}{4} - \frac{x^2}{25} = 1$$

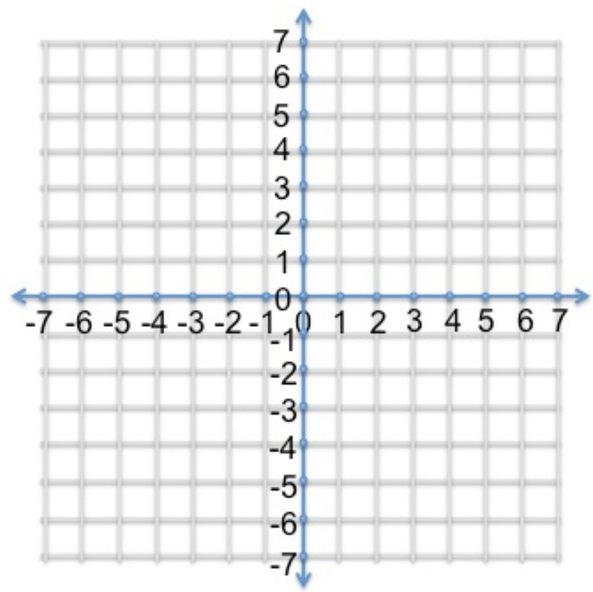
Identify form of the equation	
Identify the Center, vertices, and co-vertices	Center: Vertices: Co-Vertices:
Identify a and b	a= b=
Identify the Asymptotes	



You do with a partner in your notes.
Odd rows talk, Even rows write.
Find the vertices, co-vertices, and asymptotes of each hyperbola, and then graph.

$$\frac{x^2}{25} - \frac{y^2}{9} = 1$$

Identify form of the equation	
Identify the Center, vertices, and co-vertices	Center: V: CV:
Identify a and b	a= b=
Identify the Asymptotes Vertical $y = \pm\frac{a}{b}x$ Horizontal $y = \pm\frac{b}{a}x$	

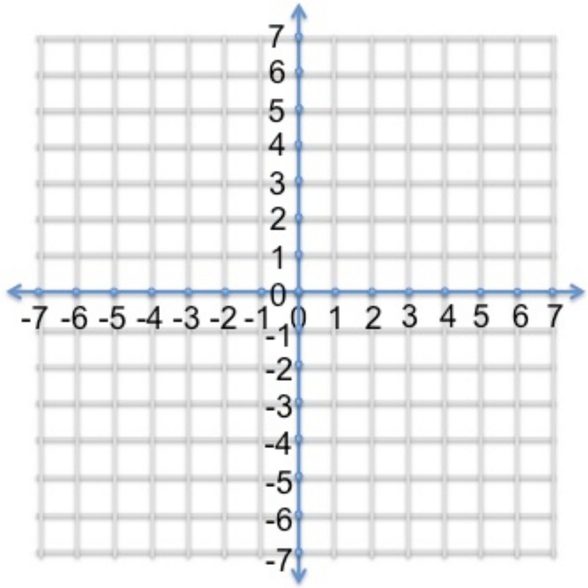


You do with a partner in your notes.
Even rows talk, Odd rows write.
Find the vertices, co-vertices, and asymptotes of each hyperbola, and then graph.

$$\frac{y^2}{4} - \frac{x^2}{1} = 1$$

Identify form of the equation	
Identify the Center,	C:

vertices, and co-vertices	V: CV:
Identify a and b	a= b=
Identify the Asymptotes Vertical $y = \pm\frac{a}{b}x$ Horizontal $y = \pm\frac{b}{a}x$	



Adding in H and K

What does H and K do again?
Compare and contrast

$$\frac{(x)^2}{4} - \frac{(y)^2}{9} = 1$$

and

$$\frac{(x-2)^2}{4} - \frac{(y+1)^2}{9} = 1$$

Standard Form for the Equation of a Hyperbola **Center at (h, k)**

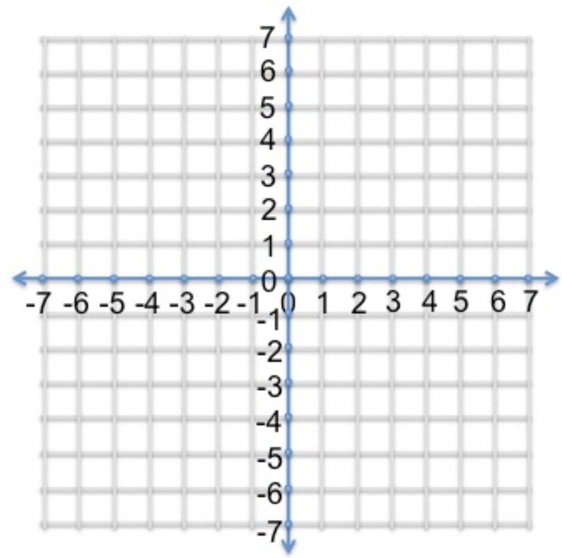
TRANSVERSE AXIS	HORIZONTAL	VERTICAL
Equation	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
Vertices	$(h+a, k), (h-a, k)$	$(h, k+a), (h, k-a)$
Foci	$(h+c, k), (h-c, k)$	$(h, k+c), (h, k-c)$
Co-vertices	$(h, k+b), (h, k-b)$	$(h+b, k), (h-b, k)$
Asymptotes	$y-k = \pm\frac{b}{a}(x-h)$	$y-k = \pm\frac{a}{b}(x-h)$

Pg. 746

Find the vertices, co-vertices, and asymptotes of each hyperbola, and then graph.

$$\frac{(x-2)^2}{4} - \frac{(y+1)^2}{9} = 1$$

Identify form of the equation	**Horizontal**
Identify a and b	$a^2 = 4$, $\boldsymbol{a = 2}$ $b^2 = 9$, $\boldsymbol{b = 3}$
Identify the center, vertices, co-vertices.	Center: (2,-1) Vertices (2+2,-1),(2-2,-1) **(4,-1),(0,-1)** Co-Vertices: (2,-1+3), (2,-1-3) **(2,2),(2,-4)**
Identify the Asymptotes Vertical: $y-k = \pm\frac{a}{b}(x-h)$ Horizontal: $y-k=\pm\frac{b}{a}(x-h)$	$\boldsymbol{y+1 = \pm\frac{3}{2}(x-2)}$

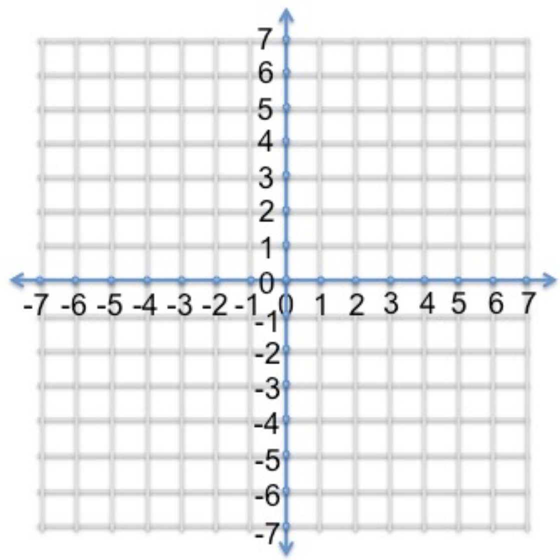


We do.

Find the vertices, co-vertices, and asymptotes of each hyperbola, and then graph.

$$\frac{(y+1)^2}{16} - \frac{(x+2)^2}{9} = 1$$

Identify form of the equation	
Identify a and b	a= b=
Identify the center, vertices, co-vertices.	Center: Vertices: Co-vertices:
Identify the Asymptotes Vertical: $y - k = \pm\frac{a}{b}\,(x - h)$ Horizontal: $y - k = \pm\frac{b}{a}\,(x - h)$	

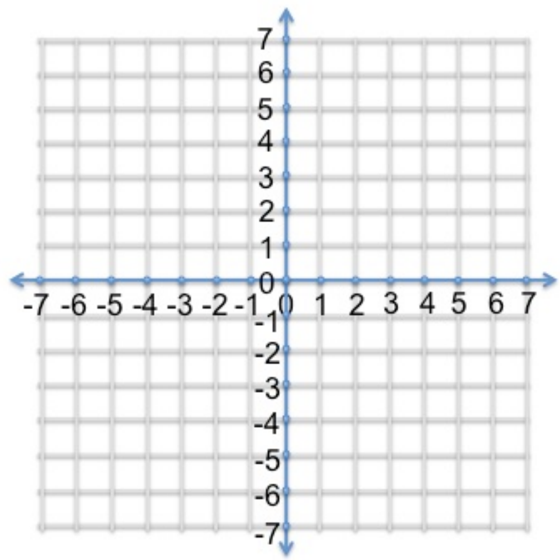


We do.

Find the vertices, co-vertices, and asymptotes of each hyperbola, and then graph.

$$\frac{(x)^2}{4} - \frac{(y-3)^2}{1} = 1$$

Identify form of the equation	
Identify a and b	a= b=
Identify the center, vertices, co-vertices.	Center: V: CV:
Identify the Asymptotes Vertical: $y-k = \pm\frac{a}{b}(x-h)$ Horizontal: $y-k = \pm\frac{b}{a}(x-h)$	

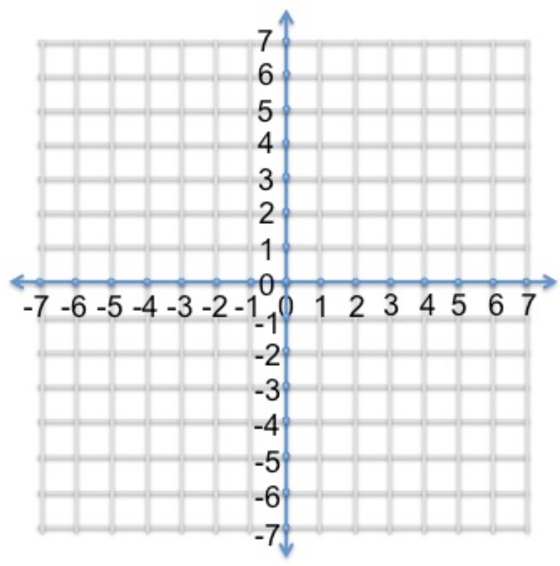


You do with your partners in your notes.
Even rows talk, odd rows write.

Find the vertices, co-vertices, and asymptotes of each hyperbola, and then graph.

$$\frac{(x-1)^2}{4} - \frac{(y-2)^2}{9} = 1$$

Identify form of the equation	
Identify a and b	a= b=
Identify the center, vertices, co-vertices.	Center: Vertices: Co-vertices:
Identify the Asymptotes Vertical: $y - k = \pm\frac{a}{b}(x-h)$ Horizontal: $y - k = \pm\frac{b}{a}(x-h)$	



Closure:

Compare and Contrast the differences between a Hyperbola and an Ellipse equation and graph. Write at least 3 aspects that are similar and 3 that are different. Be ready to share with the class

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \qquad \frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \qquad \frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$$

True or false?

Discuss with your partner about the statements. Are they true or false? Provide reasons for your answer.

$$\frac{(y+2)^2}{9} - \frac{(x-3)^2}{4} = 1$$

This is an equation for an ellipse.

This is horizontal.

a = 3.
b = 4.
The center is (-2,3).

Complete the square and identify what kind of conic it is.

Standard Forms for the Conic Sections with Center (*h*, *k*)

Circle	$(x-h)^2+(y-k)^2=r^2$	
	HORIZONTAL AXIS	**VERTICAL AXIS**
Ellipse	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$	$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$
Hyperbola	$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$	$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$
Parabola	$x-h=\frac{1}{4p}(y-k)^2$	$y-k=\frac{1}{4p}(x-h)^2$