

Completing The Square and Circles

10.2

Lesson 30 Part1

4.25.16

Warm-up

Factor

1) $x^2 + 4x + 4$

2) $x^2 - 6x + 9$

Foil

3) $(x - 4)^2$

4) $(x + 5)^2$

$$ax^2 + bx + c = 0$$

$$c = \left(\frac{b}{2}\right)^2$$

Complete the Square

1) $x^2 + 8x + \underline{\hspace{2cm}}$

$$x^2 + 8x + 16 = (x + 4)^2$$

	$x^2 + 8x + \underline{\hspace{2cm}}$
Identify B	B=8

Solve for C	$C = \left(\frac{8}{2}\right)^2 = (4)^2 = 16$
Plug back in and factor	$x^2 + 8x + 16 = (x + 4)^2$

We: Try

2) $x^2 + 6x + \underline{\hspace{2cm}} =$

	$x^2 + 6x + \underline{\hspace{2cm}}$
Identify B	$B = 6$
Solve for C	$C = \left(\frac{6}{2}\right)^2 = (3)^2 = 9$
Plug back in and factor	$x^2 + 6x + 9 = (x + 3)^2$

You Try:

4) $x^2 - 7x + \underline{\hspace{2cm}} =$

Completing the Square

$x^2 + 6x - 3 = 0$

Isolate the variables	$x^2 + 6x = 3$
Complete the square	$x^2 + 6x + \underline{\quad} = 3 + \underline{\quad}$ $x^2 + 6x + 9 = 3 + 9$
Simplify	$(x^2 + 6x + 9) = 12$ $(x + 3)^2 = 12$

Isolate the variables	$x^2 + 10x - 11 = 0$
Complete the square	
Simplify	

$$3x^2 - 24x + 53$$

Isolate the variables	$3x^2 - 24x + 53 = 0$ $3x^2 - 24x = -53$
Complete the square	$3(x^2 - 8x + \underline{\quad}) = -53$ $3(x^2 - 8x + 16) = -53 + 48$
Simplify	$3(x - 4)^2 = -5$

We Try:

$$2n^2 + 12n + 10 = 0$$

You Try:

$$7b^2 - 14b - 56 = 0$$

You Try:

$$4v^2 - 2v = 3$$

Completing the square with two variables.

$$x^2 + y^2 - 2x + 6y - 9 = 0$$

Isolate the constant	$x^2 + y^2 - 2x + 6y = 9$
Group the variables	$(x^2 - 2x) + (y^2 + 6y) = 9$
Complete the square for each variable. NOTE Whatever you do for one side you have to do for the other	$(x^2 - 2x + 1) + (y^2 + 6y + 9) = 9 + 1 + 9$
Simplify	$(x - 1)^2 + (y + 3)^2 = 19$

We Try:

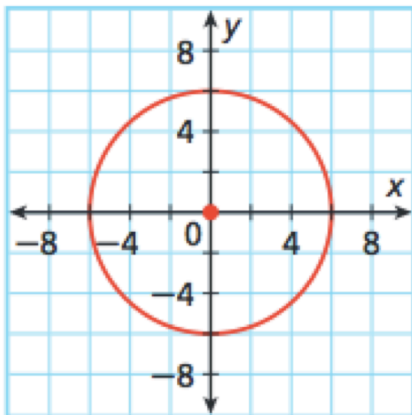
$$4x^2 - 6y^2 + 16x - 24y + 3 = 0$$

You Try:

$$4x^2 + y^2 + 8x - 8y + 16 = 0$$

Circles
10.2

Parent Function



$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = 36$$

Center: (0,0)

$$r^2 = 36$$

radius=6

Standard Form

$$(x - h)^2 + (y - k)^2 = r^2$$

Center is at (h,k)
Radius is r .

X-h	right
X+h	left
Y-k	up
Y+k	down

$$(x - 2)^2 + (y + 1)^2 = 9$$

$$h=2$$

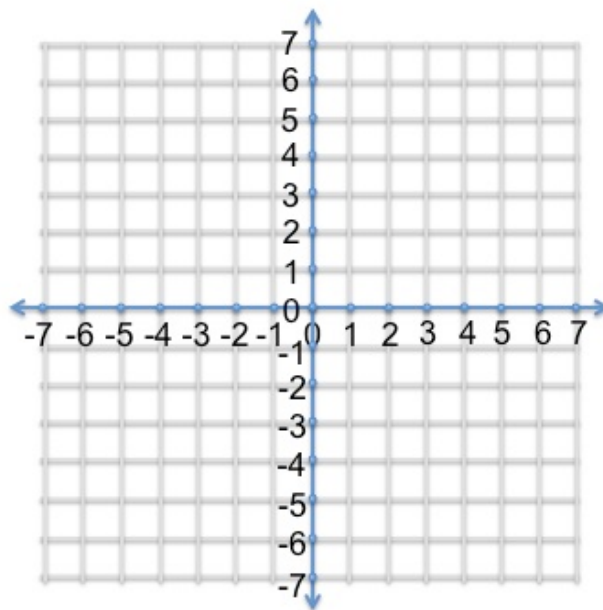
$$k=-1$$

Center at $(2,-1)$

$$r^2 = 9$$

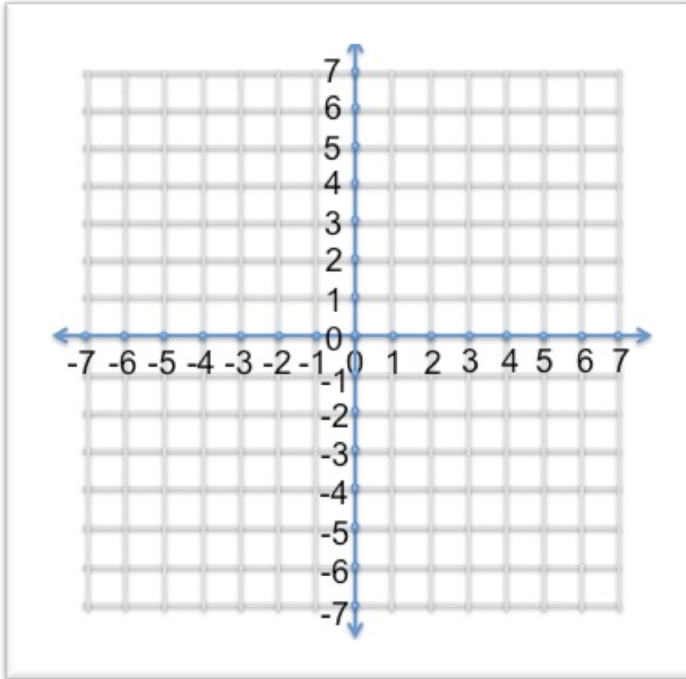
$$r=3$$

Radius is 3.



I try:
Graph the circle

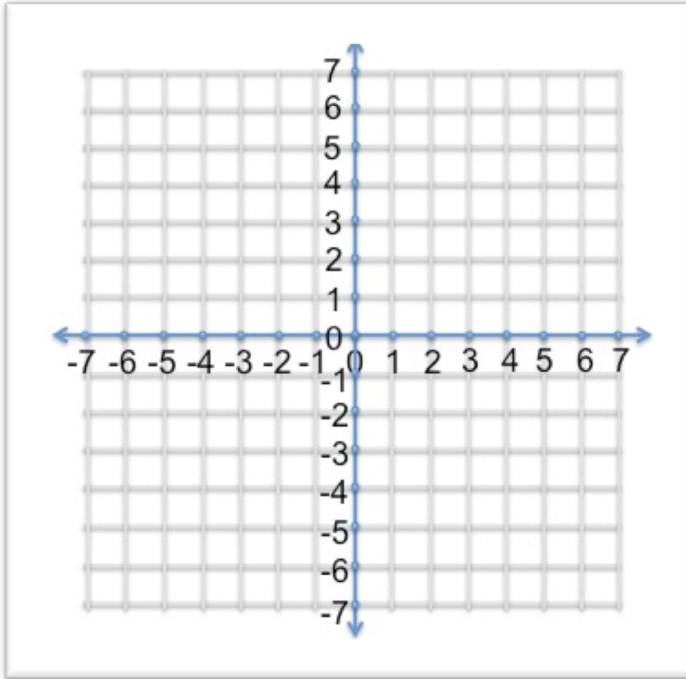
$$(x - 3)^2 + (y + 2)^2 = 25$$



We try:

Graph the circle

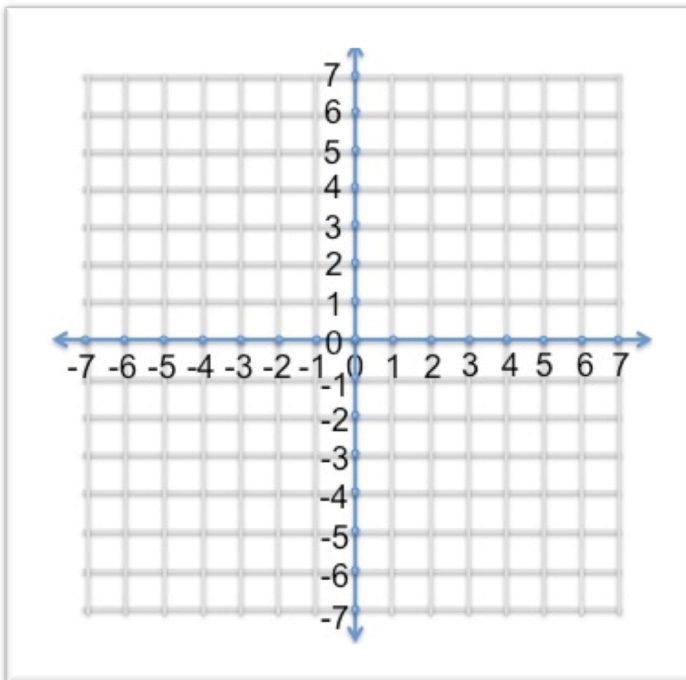
$$(x + 1)^2 + (y - 3)^2 = 4$$



We try:

Graph the circle

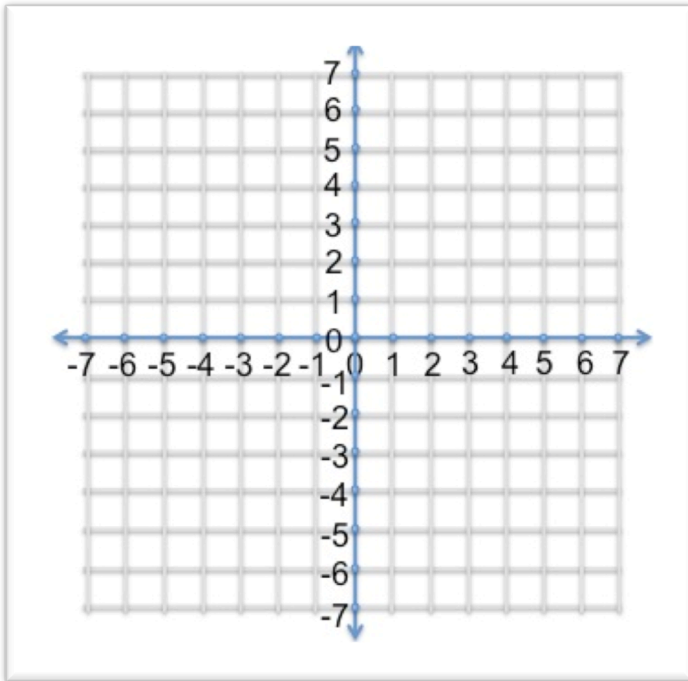
$$x^2 + (y - 1)^2 = 4$$



You Try in your notebook:

Graph the circle

$$(x - 3)^2 + (y + 1)^2 = 16$$



Write the Equation of a circle given center

$(-1, 9)$ and containing point $(2, 5)$

What do we need to find first? What can we find with a center and a point on the circle?

Reflect back to the homework due today.

We can find the radius!
center (-1,9), point (2,5)

Find Radius by using distance formula	$\sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2}$ $\sqrt{(2 - (-1))^2 + (5 - 9)^2}$ $\sqrt{(2 + 1)^2 + (-4)^2}$ $\sqrt{(3)^2 + (-4)^2}$ $\sqrt{9 + 16}$ $\sqrt{25}$ 5
Use the center and the radius to find the equation of the circle	Center (-1,9) Radius:5 $(x - h)^2 + (y - k)^2 = r^2$ $(x - (-1))^2 + (y - 9)^2 = 5^2$
Simplify	$(x + 1)^2 + (y - 9)^2 = 25$

We Try:

Write the Equation of a circle given center (2,4) and containing point (8,12)

Find the Radius using the distance formula	
Use the Center and radius to find the equation	
Simplify	

A ***Tangent*** is a line in the same plane as the circle that intersects the circle at exactly one point.

The tangent is perpendicular to the radius.

Find the Equation of the line that is tangent to the circle $25 = x^2 + y^2$ at point (3,4)

Identify the Center	Center :(0,0)
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Find the slope of the radius at the point of tangency.	$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$ $\frac{4 - 0}{3 - 0} = \frac{4}{3}$
Tangent lines are perpendicular to the radius.	$\frac{4}{3} \rightarrow -\frac{3}{4}$
Find the point and slope equation	<p>Point (3,4)</p> <p>Slope: $-\frac{3}{4}$</p> <p><i>point - slope form:</i></p> $y - y_1 = m(x - x_1)$ $y - 4 = -\frac{3}{4}(x - 3)$

We try:

Find the Equation of the line that is tangent to the circle

$$25 = (x - 1)^2 + (y + 2)^2 \text{ at point } (5, -5)$$

Identify the Center	Center :
Find the slope of the radius at the point of tangency.	$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$
Tangent lines are perpendicular to the radius.	
Find the point slope	Point :

equation.	Slope: <i>Point Slope form:</i> $y - y_1 = m(x - x_1)$
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We Try:

Find the Equation of the line that is tangent to the circle
 $25 = (x + 6)^2 + (y + 4)^2$ at point $(-9, -8)$

Identify the Center	Center :
Find the slope of the radius at the point of tangency.	Slope = $\frac{y_2 - y_1}{x_2 - x_1}$
Tangent lines are perpendicular to the radius.	
Find point- slope equation	<i>point - slope form:</i> $y - y_1 = m(x - x_1)$

You Try:

Find the Equation of the line that is tangent to the circle
 $16 = (x + 3)^2 + (y)^2$ at point $(-3, 4)$

Find the equation of the circle in standard form.

$$36 - x^2 = y^2$$