Imaginary Number Properties and Complex Graphs

9.5,9.6

3.22.16

Lesson 23

Warm-up

1) $i =$

2) $i^2 =$

3) $i^3 =$

4) $i^4 =$

5) $i^{19} =$

6) $i + 3 + 2i - 5 =$

7) $(i + 3)(2i - 4) =$

8) $What\ is\ the\ conjugate\ of\ (-2i - 3)?$

Dividing Imaginary Numbers
Example with Radical expressions.

$$\frac{5}{1+\sqrt{2}} =$$

$$\frac{5(1-\sqrt{2})}{(1+\sqrt{2})(1-\sqrt{2})} =$$

$$\frac{5(1-\sqrt{2})}{1-2} =$$

$$\frac{5(1-\sqrt{2})}{-1} =$$

$$\frac{5-5\sqrt{2}}{-1} =$$

$$-5+5\sqrt{2}$$

The same steps are taken when we divide complex numbers.

Example
Simplify:

	$\frac{2}{i}$
Multiply by Conjugate	$\frac{2(-i)}{i(-i)}$
Simplify	$\frac{-2i}{-(i^2)} =$ $\frac{-2i}{-(-1)} =$ $-\frac{2i}{1} =$ $-2i$

2)Simplify

	$\frac{3}{2-i}$
Multiply by Conjugate	$\frac{3(2+i)}{(2-i)(2+i)}$
Simplify	$\frac{3(2+i)}{(2-i)(2+i)} =$ $\frac{3(2+i)}{4-i^2} =$ $\frac{3(2+i)}{4-(-1)} =$ $\frac{3(2+i)}{4+1} =$ $\frac{3(2+i)}{5} =$ $\frac{6+3i}{5}$

We Try:

1) $\frac{6}{2+i} =$

	$\frac{6}{2+i} =$
Multiply by Conjugate	
Simplify	

2) $\frac{4+i}{-2-3i} =$

	$\frac{4+i}{-2-3i}$
Multiply by Conjugate	
Simplify	

You Try:

1) $\frac{-2}{2+i} =$

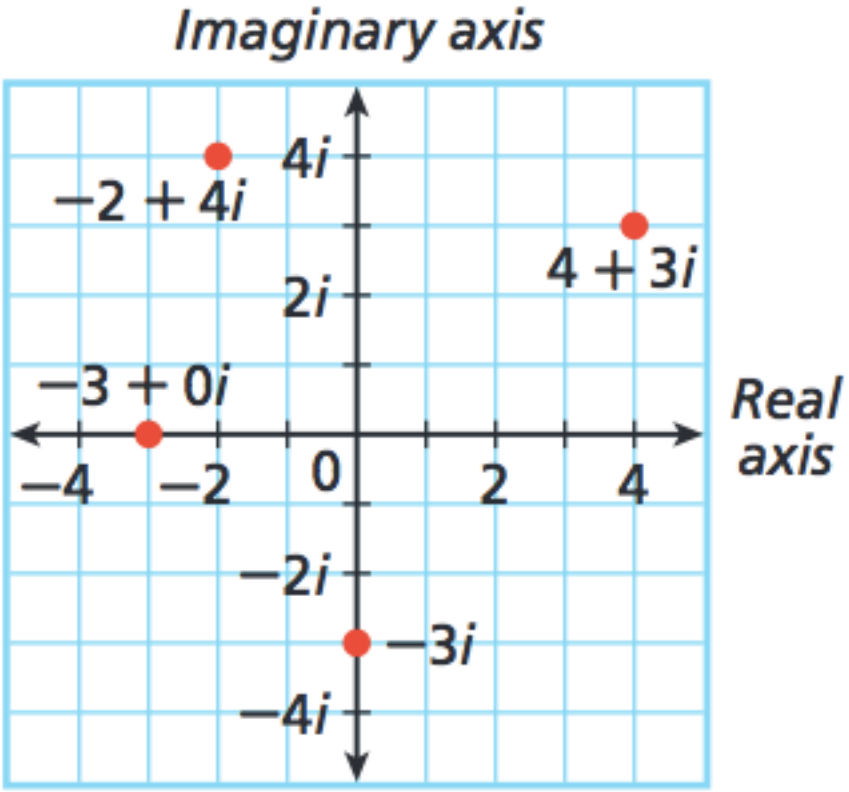
2) $\frac{4}{-5+2i} =$

3) $\frac{5i}{-3+2i} =$

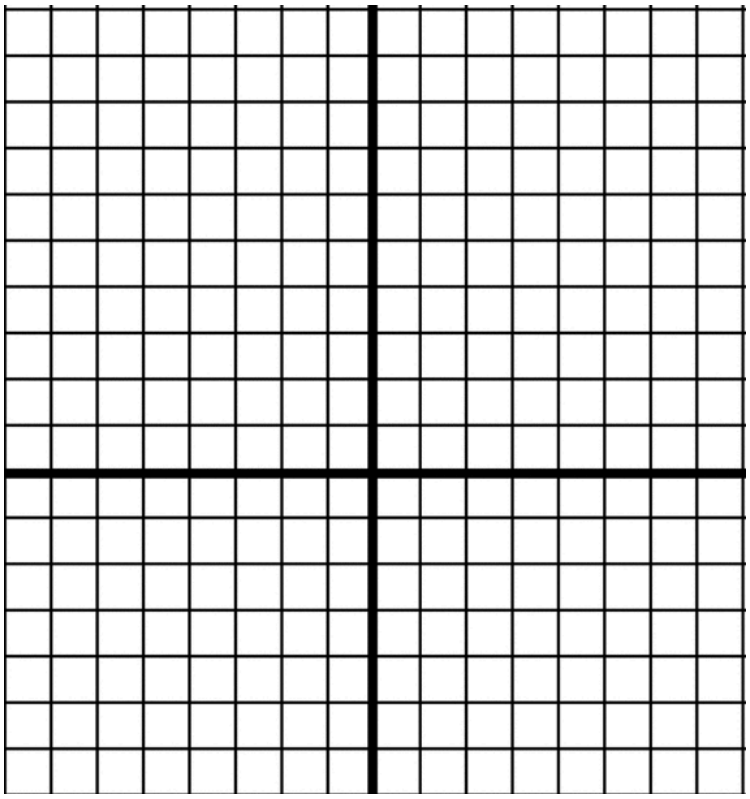
Graphing Complex Numbers

The **Complex Plane** is a set of coordinate axes in which the horizontal axis represents real numbers and the vertical axis represents imaginary numbers.

Examples

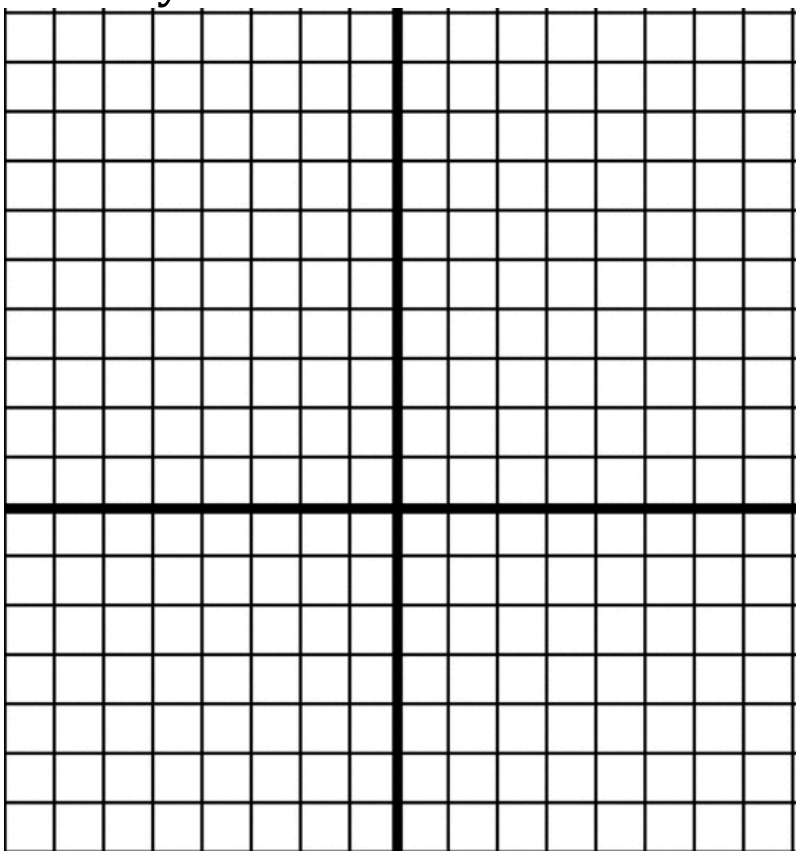


We Try:



1) 4+3i 2)-2-i 3) -i

You Try on whiteboards:



Plot these all on the whiteboard.

1) -3+2i 2)-3 3)

In the case of a complex number, r, represents the absolute value, or ***modulus***, of the complex number.

$$r = \sqrt{(a)^2 + (b)^2}$$

The angle, θ, is called the **amplitude or argument,** of the complex number.

$$\theta = \tan^{-1}\frac{b}{a} \; if \; a > 0$$

$$\theta = \tan^{-1}\frac{b}{a} + \; \pi \quad if \; b < 0$$

Given a complex number $a + bi$, the polar form is written as $\mathbf{r}(cos\theta + \mathbf{i}sin\theta)$.

$\mathbf{r}(cos\theta + \mathbf{i}sin\theta)$ can be shortened to $\boldsymbol{rcis\theta}$.

I Try:
Express the complex number in polar form

$$-3 + 4i$$

Find the modulus, r	
Find the amplitude, θ	
Plug into $\mathbf{r}(cos\theta + \mathbf{i}sin\theta)$	

We Try:

Express the complex number in polar form

$$1 + \sqrt{3}i$$

Find the modulus, r	
Find the amplitude, θ	
Plug into $\mathbf{r}(cos\theta + \mathbf{i}sin\theta)$	

You Try:
Express the complex number in polar form

$$2 - 3i$$

Find the modulus, r	

Find the amplitude, θ	
Plug into $\mathbf{r}(cos\theta + \mathbf{i}sin\theta)$	

I Try:

Graph $4(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6})$

Find the polar coordinates (r, θ)	$r =$ $\theta =$
Graph the point	
Converting to Rectangular $x = rcos\theta$ $y = rsin\theta$	$x =$ $y =$

We Try:

Graph $2(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6})$ and write in rectangular form.

Find the polar coordinates (r,θ)	$r=$ $\theta=$
Graph the point	
Converting to Rectangular $x=rcos\theta$ $y=rsin\theta$	$x=$ $y=$

You Try:

Graph $2(\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3})$ and write in rectangular form

Find the polar coordinates (r,θ)	$r=$ $\theta=$
Graph the point	
Converting to Rectangular $x=rcos\theta$ $y=rsin\theta$	$x=$ $y=$

Exit Ticket

1) Graph $3(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$ and write in rectangular form.
2) Simplify $\frac{3}{2-i}$.
3) Express the complex number in polar form $-2 + 3i$