

Lesson 9  
9.21.15  
(section 3.1 and 3.2)  
Function Symmetry.

A function has a graph that is symmetrical around the origin if and only if  $f(-x) = -f(x)$  for all  $x$  in the domain of  $f$ .

THE TEST: if  $(x, y) \in S$ , does  $(-x, -y) \in S$ ?  
 $\in$  means *belongs to*

I try:

Is  $f(x) = x^5$  symmetrical around the origin?

Solve it algebraically.

<i>Given</i>	$f(x) = x^5$	
Solve for $f(-x)$ and $-f(x)$	$f(-x) = (-x)^5$	$-f(x) = -x^5$
Check Does $f(-x) = -f(x)$ ?	Does $(-x)^5 = -x^5$ $-x^5 = -x^5$ YES $f(x) = x^5$ is symmetrical around the origin.	

We Try:

Is  $f(x) = x - 5$  symmetrical around the origin?

Solve it algebraically.

<i>Given</i>	$f(x) = x - 5$	
Solve for $f(-x)$ and $-f(x)$	$f(-x) =$	$-f(x) =$
Check Does $f(-x) = -f(x)$ ?		

You Try with your partner on whiteboards:  
Left column talks, right column writes.

Is  $f(x) = x - 5$  symmetrical around the origin?

Solve it algebraically.

<i>Given</i>	$f(x) = x^2 - 3$	
Solve for $f(-x)$ and $-f(x)$	$f(-x) =$	$-f(x) =$
<i>Check</i> Does $f(-x) = -f(x)$ ?		

Does the graph have symmetry with respect to..

Symmetry	Test	Example
x-axis	$(x,y) \Rightarrow (x,-y)$	<p>Graph of the parabola <math>x = y^2 - 4</math> opening to the right. The x-axis is highlighted in red and labeled <math>y = 0</math>. The origin is <math>O</math>. Two points are marked: <math>(2, \sqrt{6})</math> and <math>(2, -\sqrt{6})</math>.</p>
y-axis	$(x,y) = (-x,y)$	<p>Graph of the parabola <math>y = -x^2 + 12</math> opening downwards. The y-axis is highlighted in red and labeled <math>x = 0</math>. The origin is <math>O</math>. Two points are marked: <math>(-2, 8)</math> and <math>(2, 8)</math>.</p>
$y = x$	$(x,y) = (y,x)$	<p>Graph of the hyperbola <math>xy = 6</math>. A dashed red line <math>y = x</math> is shown. Two points are marked: <math>(2, 3)</math> and <math>(3, 2)</math>.</p>
$y = -x$	$(x,y) = (-y,-x)$	<p>Graph of the ellipse <math>17x^2 + 16xy + 17y^2 = 225</math>. A dashed red line <math>y = -x</math> is shown. Two points are marked: <math>(4, -1)</math> and <math>(1, -4)</math>.</p>

I Try:

Determine whether the graph of  $y = x^2 + 5$  is symmetric with respect to the x-axis, y-axis,  $y=x$ , and  $y=-x$ .

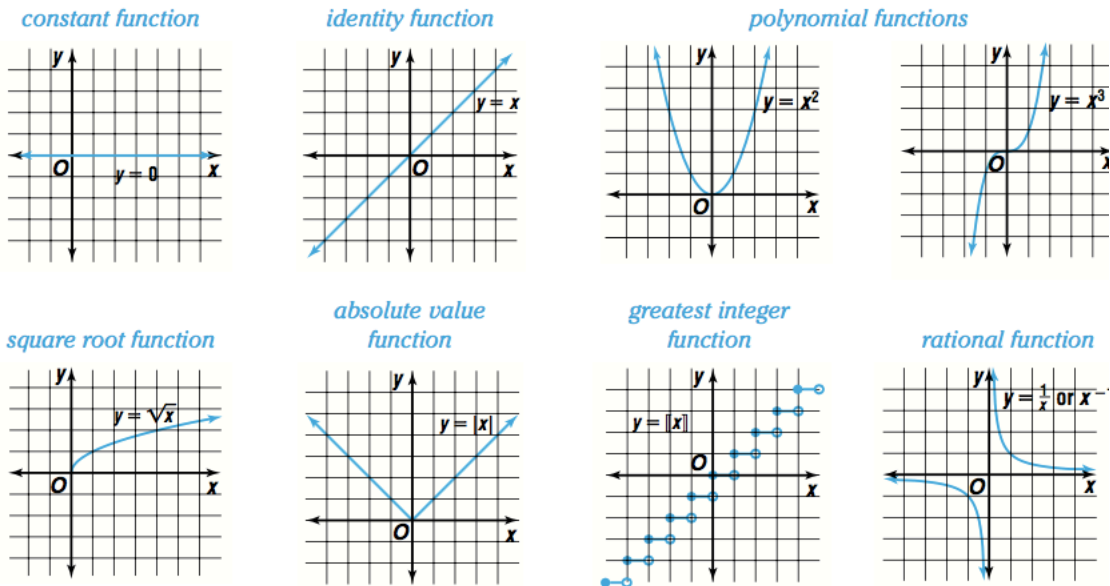
Symmetry	Test	$y = x^2 + 5$
Rearrange		$y - x^2 = 5$
x-axis	$(x,y) \Rightarrow (x,-y)$	$y - x^2 \neq -y - x^2$ No
y-axis	$(x,y) \Rightarrow (-x,y)$	$y - x^2 = y - (-x)^2$ Yes
$y=x$	$(x,y) \Rightarrow (y,x)$	$y - x^2 \neq x - y^2$ No
$y=-x$	$(x,y) \Rightarrow (-y,-x)$	$y - x^2 \neq -x - (-y)^2$ No

We Try:

Determine whether the graph of  $y = -8x$  is symmetric with respect to the x-axis, y-axis,  $y=x$ , and  $y=-x$ .

Symmetry	Test	$y = -\frac{8}{x}$
Rearrange		
x-axis	$(x,y) = (x,-y)$	
y-axis	$(x,y) = (-x,y)$	
$y=x$	$(x,y) = (y,x)$	
$y=-x$	$(x,y) = (-y,-x)$	

## 3.2 Families of Graphs Pg 139

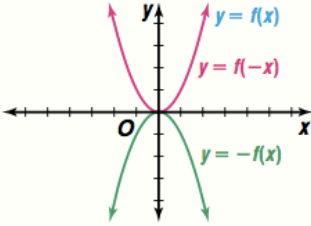
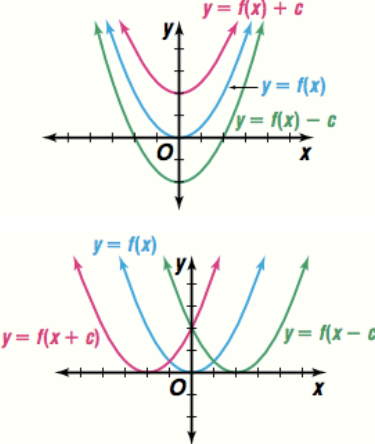
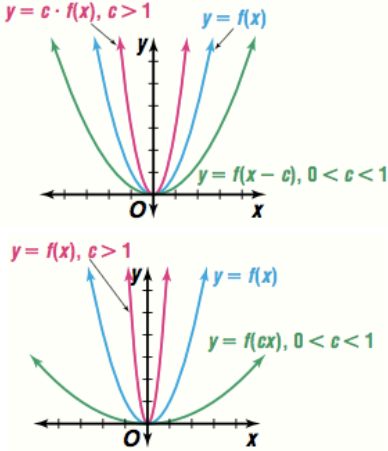


Closure:

Discuss with your partner. Be prepared to share  
What are the different tests for symmetry?

Test  $xy = 3$

*for the different symmetries.*

Change to the Parent Function $y = f(x)$ , $c > 0$	Change to Parent Graph	Examples
<p><b>Reflections</b></p> <p><math>y = -f(x)</math>  <math>y = f(-x)</math></p>	<p>Is reflected over the x-axis.  Is reflected over the y-axis.</p>	
<p><b>Translations</b></p> <p><math>y = f(x) + c</math>  <math>y = f(x) - c</math></p> <p><math>y = f(x + c)</math>  <math>y = f(x - c)</math></p>	<p>Translates the graph c units up.  Translates the graph c units down.</p> <p>Translates the graph c units left.  Translates the graph c units right.</p>	
Change to the Parent Function $y = f(x)$ , $c > 0$	Change to Parent Graph	Examples
<p><b>Dilations</b></p> <p><math>y = c \cdot f(x)</math>, <math>c &gt; 1</math>  <math>y = c \cdot f(x)</math>, <math>0 &lt; c &lt; 1</math></p> <p><math>y = f(cx)</math>, <math>c &gt; 1</math>  <math>y = f(cx)</math>, <math>0 &lt; c &lt; 1</math></p>	<p>Expands the graph vertically.  Compresses the graph vertically.</p> <p>Compresses the graph horizontally.  Expands the graph horizontally.</p>	



We Try:

18) Describe how the graphs of  $f(x)$  and  $g(x)$  are related.

$$f(x) = \lceil x \rceil + 1$$

$$g(x) = -\lceil x \rceil - 1$$