Determinants Of Matrices
Lesson 8
9/10/15

Each Square Matrix has a determinant.

The Determinant of $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ is denoted by $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix}$.

The value of $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ is $a_1 b_2 - a_2 b_1$.

I Try:

Find the value of $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} =$

Given	$\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix}$
Plug into formula	$2(5)-4(3).$
Simplify	10-12= -2 Determinant = -2

We Try:

Find the value of $\begin{vmatrix} -1 & 5 \\ .4 & 0 \end{vmatrix} =$

Given	$\begin{vmatrix} -1 & 5 \\ .4 & 0 \end{vmatrix}$
Plug into formula	
Simplify	

You Try on your whiteboards.

1) $\begin{vmatrix} -5 & 6 \\ 2 & 3 \end{vmatrix}$

2) $\begin{vmatrix} \frac{1}{2} & 9 \\ \frac{1}{3} & 4 \end{vmatrix}$

Find the determinant of a $3x3$ matrix

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

2 **Find the value of** $\begin{vmatrix} -4 & -6 & 2 \\ 5 & -1 & 3 \\ -2 & 4 & -3 \end{vmatrix}$.

$$\begin{vmatrix} -4 & -6 & 2 \\ 5 & -1 & 3 \\ -2 & 4 & -3 \end{vmatrix} = -4 \begin{vmatrix} -1 & 3 \\ 4 & -3 \end{vmatrix} - (-6) \begin{vmatrix} 5 & 3 \\ -2 & -3 \end{vmatrix} + 2 \begin{vmatrix} 5 & -1 \\ -2 & 4 \end{vmatrix}$$

$$= -4(-9) + 6(-9) + 2(18)$$
$$= 18$$

OR

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$aei + bfg + cdh - ceg - fha - ibd$$

$$\begin{vmatrix} -4 & -6 & 2 \\ 5 & -1 & 3 \\ -2 & 4 & -3 \end{vmatrix}$$

$$-4(-1)(-3) + (-6)(3)(-2) + 2(5)(4) - (2)(-1)(-2) - (-3)(4)(-4) - (3)(-6)(5)$$

$$-12 + 36 + 40 - 4 + 48 - 90 = 18$$

We Try:

$$\begin{vmatrix} -1 & 1 & 2 \\ -3 & 0 & 3 \\ -2 & -4 & 4 \end{vmatrix}$$

You Try on your whiteboards:

$$\begin{vmatrix} 0 & 1 & 2 \\ 2 & 0 & 3 \\ 1 & -1 & 3 \end{vmatrix}$$

Cramer's Rule
Solve for x,y, and z.

$$2x + y + z = 3$$
$$x - y - z = 0$$
$$x + 2y + z = 0$$

system of equations	coefficient matrix's determinant	answer column	D_x: coefficient determinant with answer-column values in x-column
$2x + 1y + 1z = 3$ $1x - 1y - 1z = 0$ $1x + 2y + 1z = 0$	$D = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & 2 & 1 \end{vmatrix}$	$\begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$	$D_x = \begin{vmatrix} 3 & 1 & 1 \\ 0 & -1 & -1 \\ 0 & 2 & 1 \end{vmatrix}$

Similarly, D_y and D_z would then be:

$$D_y = \begin{vmatrix} 2 & 3 & 1 \\ 1 & 0 & -1 \\ 1 & 0 & 1 \end{vmatrix} \qquad D_z = \begin{vmatrix} 2 & 1 & 3 \\ 1 & -1 & 0 \\ 1 & 2 & 0 \end{vmatrix}$$

Evaluating each determinant, we get:

$$D = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & 2 & 1 \end{vmatrix} = (-2)+(-1)+(2) - (-1)-(-4)-(1) = 3$$

$$D_x = \begin{vmatrix} 3 & 1 & 1 \\ 0 & -1 & -1 \\ 0 & 2 & 1 \end{vmatrix} = (-3)+(0)+(0) - (0)-(-6)-(0) = -3+6 = 3$$

$$D_y = \begin{vmatrix} 2 & 3 & 1 \\ 1 & 0 & -1 \\ 1 & 0 & 1 \end{vmatrix} = (0)+(-3)+(0) - (0)-(0)-(3) = -3-3 = -6$$

$$D_z = \begin{vmatrix} 2 & 1 & 3 \\ 1 & -1 & 0 \\ 1 & 2 & 0 \end{vmatrix} = (0)+(0)+(6) - (-3)-(0)-(0) = 6+3 = 9$$

Cramer's Rule says that $x = D_x \div D$, $y = D_y \div D$, and $z = D_z \div D$. That is:

$$x = {}^{3}/_{3} = 1, \; y = {}^{-6}/_{3} = -2, \text{ and } \; z = {}^{9}/_{3} = 3$$

(1,-2,3)

We Try:

$$4x = -8z$$
$$3x - 2y + z = 0$$
$$-2x + y - z = -1$$

Given	$4x = -8z$ $3x - 2y + z = 0$ $-2x + y - z = -1$
Lineup the equations	$4x + 0y + 8z = 0$ $3x - 2y + z = 0$ $-2x + y - z = -1$

Create the matrices	
Find the determinants.	
Use Cramer's Rule $x = \frac{Dx}{D}, y = \frac{Dy}{D}, z = \frac{Dz}{D},$	

$$7x + 5y + z = 0$$
$$-x + 3y + 2z = 16$$
$$x - 6y - z = -18$$

Finding the inverse

Given $A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$ and $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \neq 0$

$$A^{-1} = \frac{1}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \begin{bmatrix} b_2 & -b_1 \\ -a_2 & a_1 \end{bmatrix}$$

If the determinant is 0 there is no inverse.

$$AA^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is the identity matrix.

I Try:

Find the inverse of $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix}$.

Given	$\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix}$
Find the determinant.	$2(5)-4(3).$ $10-12 = -2$ $\text{Det} = -2$
Plug determinant into formula	$\frac{1}{-2}\begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix}$
Simplify	$\begin{bmatrix} -\frac{5}{2} & \frac{3}{2} \\ 2 & -1 \end{bmatrix}$

We Try:
Find the inverse of

$$\begin{bmatrix} -1 & 5 \\ .4 & 0 \end{bmatrix}$$

Given	$\begin{bmatrix} -1 & 5 \\ .4 & 0 \end{bmatrix}$
Find Determinant	
Plug determinant into formula	
Simplify	

You Try on your whiteboards:
Find the inverse of

$$\begin{bmatrix} 4 & 2 \\ 1 & 2 \end{bmatrix}$$

Solve the system of equations using matrices.

$$2x + 3y = -17$$
$$x - y = 4$$

Given	$2x + 3y = -17$ $x - y = 4$
Create the matrices	$\begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -17 \\ 4 \end{bmatrix}$
Find the inverse of the first matrix	$\frac{1}{\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix}}\begin{bmatrix} -1 & -3 \\ -1 & 2 \end{bmatrix} = -\frac{1}{5}\begin{bmatrix} -1 & -3 \\ -1 & 2 \end{bmatrix}$ $= \begin{bmatrix} \frac{1}{5} & \frac{3}{5} \\ \frac{1}{5} & -\frac{2}{5} \end{bmatrix}$
Multiply both sides by the inverse.	$\begin{bmatrix} \frac{1}{5} & \frac{3}{5} \\ \frac{1}{5} & -\frac{2}{5} \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & \frac{3}{5} \\ \frac{1}{5} & -\frac{2}{5} \end{bmatrix}\begin{bmatrix} -17 \\ 4 \end{bmatrix}$
Simplify	$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & \frac{3}{5} \\ \frac{1}{5} & -\frac{2}{5} \end{bmatrix}\begin{bmatrix} -17 \\ 4 \end{bmatrix}$

	$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$ (-1,5)
Given	$2x + 3y = -17$ $x - y = 4$
Create the matrices	
Find the inverse of the first matrix	
Multiply both sides by the inverse.	
Simplify	