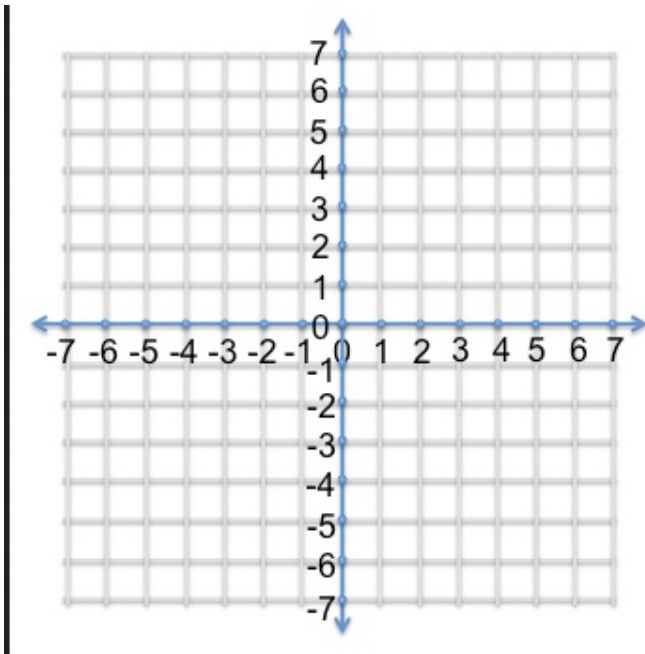


Warm-up 10.5.15 -10.6.15  
Lesson 11 Rational Functions  
Fill the table and Graph  $y = \frac{1}{x}$



X	Y
-4	
-2	
-1	
-.5	
0	
.5	
1	
2	
3	

### Rational Functions

A ***Rational Function*** is a function whose rule can be written as a ratio of two polynomials.

Rational Functions have Asymptotes.

**Asymptotes** are lines that the graph approaches as  $x$  becomes very large or small.

**Horizontal Asymptote** are when  $y =$

**Vertical Asymptote** are when  $x =$

What were the Asymptotes for  $y = \frac{1}{x}$  ?

Consider this to be a parent function. Our goal is to find out what the transformations are

Describe the transformations. State the domain and range.

Identify the asymptotes.

As  $x \rightarrow \infty, y \rightarrow$

As  $x \rightarrow -\infty, y \rightarrow$

1)  $y = \frac{1}{x-2}$

2)  $y = \frac{1}{x} - 3$

$$3) y = \frac{1}{x+2} + 1$$

$$4) y = \frac{3}{x}$$

$$5) y = \frac{-1}{x}$$

$|a| \rightarrow$  vertical stretch or compression factor  
 $a < 0 \rightarrow$  reflection across the  $x$ -axis

$$f(x) = \frac{a}{x-h} + k$$

$k \rightarrow$  vertical translation  
down for  $k < 0$ ; up for  $k > 0$

$h \rightarrow$  horizontal translation  
left for  $h < 0$ ; right for  $h > 0$

We Try:

Describe the transformations. State the domain and range.

Identify the asymptotes.

As  $x \rightarrow \infty, y \rightarrow$

As  $x \rightarrow -\infty, y \rightarrow$

$$1) y = \frac{1}{x-5} + 6$$

$$2) y = \frac{-1}{x} - 4$$

$$3) y = \frac{4}{x-3} + 2$$

You try:

Describe the transformations. State the domain and range.

Identify the asymptotes.

As  $x \rightarrow \infty, y \rightarrow$

As  $x \rightarrow -\infty, y \rightarrow$

Left talk, Right write on whiteboard.

$$1) y = \frac{1}{x-2} + 4$$

Right talk, Left write in notebook

$$2) y = \frac{-2}{x}$$

$$3) y = \frac{3}{x+2} + 1$$

Lesson 11P1 HW Pg. 185 #1,5,7,8,9,21-27

**Closure: Stand up hand up!**

**Older person:** Explain how you would shift  $y = \frac{1}{x}$  down 3 units.

**Younger person:** Explain how you would shift  $y = \frac{1}{x}$  left 2 units.

Find another partner.

**Taller person:** Explain how you would shift  $y = \frac{1}{x} + 2$  up 1 unit.

**Shorter person:** Explain how you would shift  $y = \frac{1}{x-2}$  right 1 unit.

Warm-up

10.6.15

1) *Factor*  $x^2 + 5x + 6$

2) *Divide:*  $(n^3 + 7n^2 + 14n + 3) \div (n + 2)$

3) *Divide:*  $(v^3 - 2v^2 - 14v - 5) \div (v + 3)$

## Graphing Rational Functions with Vertical Asymptotes

Identify the zeros and vertical asymptotes of  $f(x) = \frac{x^2 - 2x - 3}{x - 2}$ .  
Then graph.

**Step 1** Find the zeros and vertical asymptotes.

$$f(x) = \frac{(x + 1)(x - 3)}{x - 2}$$

*Factor the numerator.*

Zeros:  $-1$  and  $3$

*The numerator is 0 when  $x = -1$  or  $x = 3$ .*

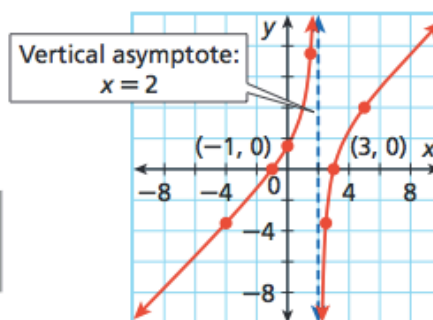
Vertical asymptote:  $x = 2$

*The denominator is 0 when  $x = 2$ .*

**Step 2** Graph the function.

Plot the zeros and draw the asymptote. Then make a table of values to fill in missing points.

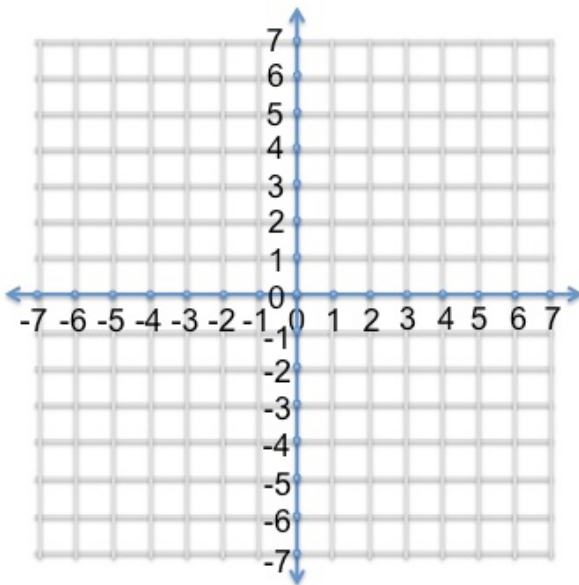
$x$	$-4$	$-1$	$0$	$1.5$	$2.5$	$3$	$5$
$y$	$-3.5$	$0$	$1.5$	$7.5$	$-3.5$	$0$	$4$



$$y = \frac{x^2 + 5x + 6}{x - 1}$$

Rational Function	$y = \frac{x^2 + 5x + 6}{x - 1}$
Factor	$y = \frac{(x + 3)(x + 2)}{x - 1}$
Identify 0's,	$X = -3, -2$
Identify Vertical Asymptotes (undefined values)	$X = 1$

Create table and plot points around the vertical asymptote



x	y
-5	
0	
.5	
1	
1.5	
3	
5	

We try:

Graph and identify the zeroes and vertical asymptotes of

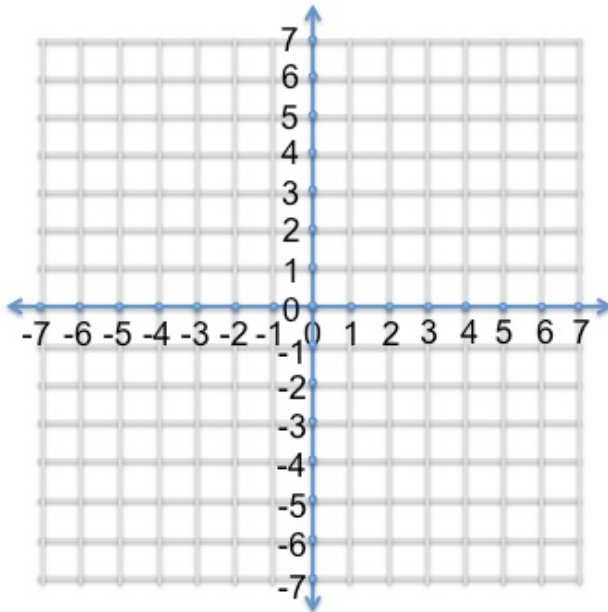
$$y = \frac{x^2 + 7x + 6}{x + 3}$$

Rational Function	$y = \frac{x^2 + 7x + 6}{x + 3}$
Factor	
Identify 0's,	



Identify Vertical Asymptotes  
(undefined values)

Create table and plot points



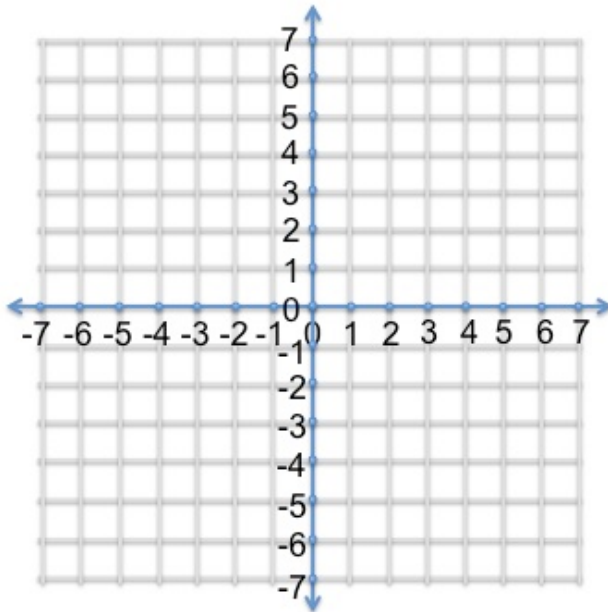
x	y

Graph and identify the zeroes and vertical asymptotes of  
 $y = \frac{x^2 - 1}{x - 2}$

Rational Function	$y = \frac{x^2 - 1}{x - 2}$
Factor	
Identify 0's,	

Identify Vertical Asymptotes  
(undefined values)

Create table and plot points



x	y

You try:  
Graph and identify the zeroes and  
vertical asymptotes of  $y = \frac{x^2+4x}{x+2}$ .

## The Special Cases!

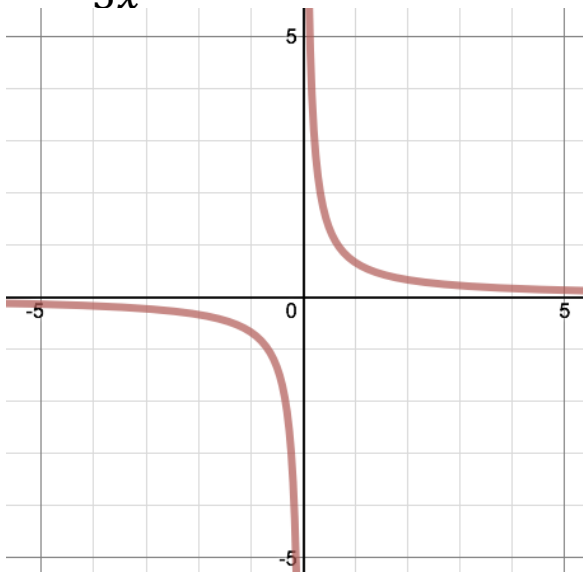
$$\text{Let } f(x) = \frac{p(x)}{q(x)}$$

### Case #1

If the degree of **p** < degree of **q**, the horizontal asymptote is  $y = 0$ .

Example:

$$y = \frac{2x}{3x^2}$$



## Case #2

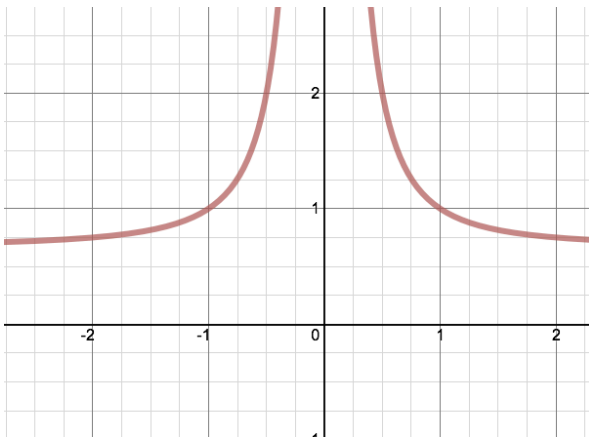
If the degree of  $\mathbf{p}$  = degree of  $\mathbf{q}$ , the horizontal asymptote is

$$y = \frac{\text{leading coefficient of } p}{\text{leading coefficient of } q}$$

## Example

$$y = \frac{2x^2+1}{3x^2}$$

horizontal asymptote :  $y = \frac{2}{3}$

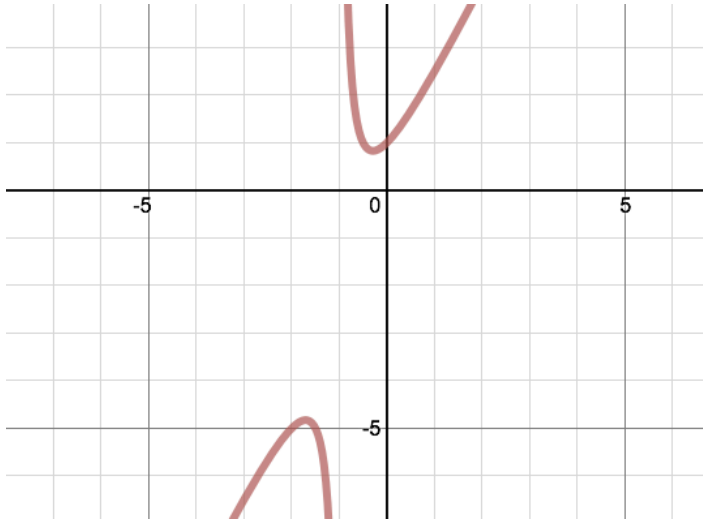


### Case #3

If the degree of  $p = (\text{degree of } q) + 1$ , there is a slant asymptote.

Example

$$y = \frac{2x^2 + 2x + 1}{x + 1}$$



To find the slant asymptote, divide  $p(x)$  by  $q(x)$  and graph the quotient without remainders.

$$y = \frac{2x^2 + 2x + 1}{x + 1}$$

$$(2x^2 + 2x + 1) \div (x + 1)$$

We try:

Identify the zeroes and asymptotes of each function.

$$1) y = \frac{x^2 - 6x + 5}{x - 2}$$

$$2) y = \frac{5x + 2}{x + 1}$$

You try:

Identify the zeroes and asymptotes of each function.

Left Talk, Right Write in notebook.

$$y = \frac{x^2 - 5x - 6}{x + 2}$$

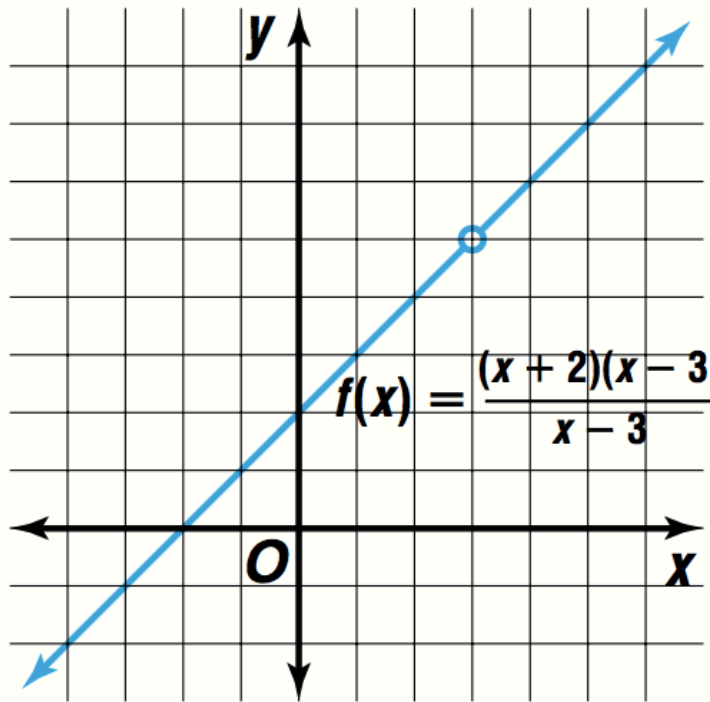
Right Talk, Left write in notebook.

$$y = \frac{2x^2}{x^2 - 4}$$

HOLES

What is a hole?

What do you think a hole in a graph means?



$$f(x) = \frac{(x+2)(x-3)}{x-3} = \frac{x^2-x-6}{x-3}$$

Where was the hole located? What do you think the relationship is?



A hole is when there is a common factor on the numerator and denominator of a rational fraction.

I Do:

$$f(x) = \frac{(x^2 + 5x + 4)}{x + 4}$$

Factor	$f(x) = \frac{(x + 4)(x + 1)}{x + 4}$
Identify hole by setting canceled factor to 0	$x + 4 = 0$ $x = -4$
Cancel out factors	$f(x) = (x + 1)$
Graph the leftover function and insert hole.	

We Do:  $f(x) = \frac{-x^2 + x}{x - 1}$

Factor	$f(x) =$
Identify hole by setting canceled factor to 0	

Cancel out factors	
Graph the leftover function and insert hole.	

Closure

Exit Slip

Go find your 6 o'clock partner. On ONE piece of paper graph and label the zeros, asymptotes, and holes.

$$f(x) = \frac{(x^2 + 6x + 7)}{x + 1}$$