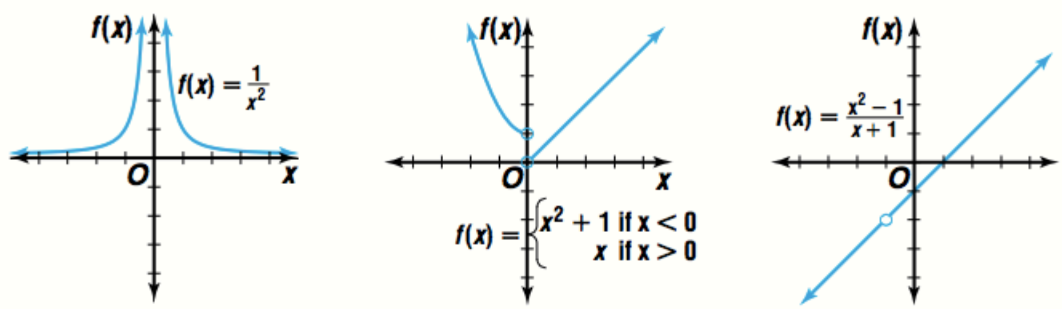
Continuity and End Behavior
10.15.15
Lesson 14

- **Infinite discontinuity** means that $|f(x)|$ becomes greater and greater as the graph approaches a given x-value.
- **Jump discontinuity** indicates that the graph stops at a given value of the domain and then begins again at a different range value for the same value of the domain.
- When there is a value in the domain for which the function is undefined, but the pieces of the graph match up, we say the function has **point discontinuity.**

Graphs of Discontinuous Functions



Infinite Discontinuity | Jump Discontinuity | Point Discontinuity

A ***<u>Continuous</u>*** function is a function that is not discontinuous.

What are some continuous functions that we have learned about before?

Continuity Test

A function is continuous at $x = c$ if it satisfies the following conditions:
(1) the function is defined at c; in other words, $f(c)$ exists;
(2) the function approaches the same y-value on the left and right sides of $x = c$; and
(3) the y-value that the function approaches from each side is $f(c)$.

Determine whether each function is continuous at the given x-value.

a. $f(x) = 3x^2 + 7;\ x = 1$

Check the three conditions in the continuity test.

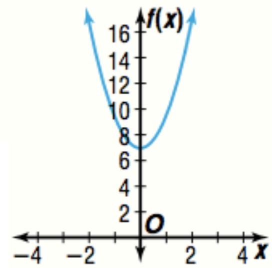
(1) The function is defined at $x = 1$. In particular, $f(1) = 10$.

(2) The first table below suggests that when x is less than 1 and x approaches 1, the y-values approach 10. The second table suggests that when x is greater than 1 and x approaches 1, the y-values approach 10.

x	$y = f(x)$
0.9	9.43
0.99	9.9403
0.999	9.994003

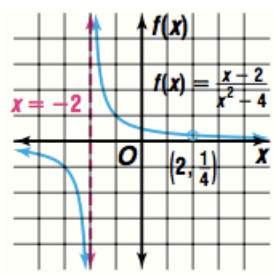
x	$y = f(x)$
1.1	10.63
1.01	10.0603
1.001	10.006003

(3) Since the y-values approach 10 as x approaches 1 from both sides and $f(1) = 10$, the function is continuous at $x = 1$. This can be confirmed by examining the graph.



b. $f(x) = \dfrac{x - 2}{x^2 - 4};\ x = 2$

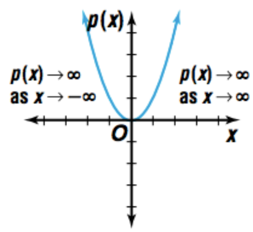
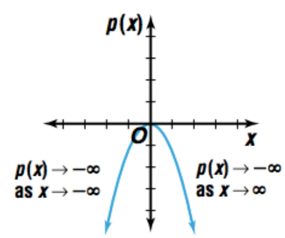
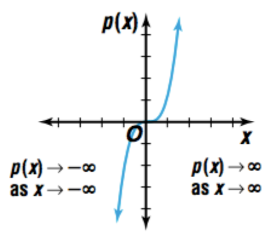
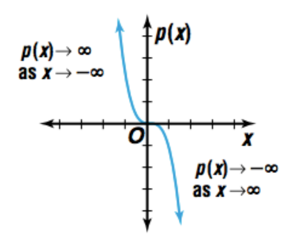
Start with the first condition in the continuity test. The function is not defined at $x = 2$ because substituting 2 for x results in a denominator of zero. So the function is discontinuous at $x = 2$.
This function has point discontinuity at $x = 2$.



End behavior!

End Behavior of Polynomial Functions

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \dots + a_2 x^2 + a_1 x + a_0,\ n > 0$$

a_n: positive, n: even	a_n: negative, n: even
$p(x) = x^2$ 	$p(x) = -x^2$ 
a_n: positive, n: odd	**a_n: negative, n: odd**
$p(x) = x^3$ 	$p(x) = -x^3$ 

The **leading term** is the term with the highest exponent.

I Try:
Determine the end behavior.

$$y = x - 3x^2 - 2$$

Put in standard form	$y = -3x^5 + x - 2$
Identify leading term	$-3x^5$
Identify $\boldsymbol{a}$ *and* $\boldsymbol{n}$	$a = -3, negative$ $n = 5, odd$
Reference chart and state end behavior.	$as\ x \to \infty, y \to -\infty$ $as\ x \to -\infty, y \to \infty$

We Try:
Determine the end behavior.

$$y = x^2 + 3x^4 - 2x - 15$$

Put in standard form	
Identify leading term	
Identify $\boldsymbol{a}$ *and* $\boldsymbol{n}$	a $n =$
Reference chart and state end behavior.	$as\ x \to \infty, y \to$ $as\ x \to -\infty, y \to$

We Try:
Determine the end behavior.

$$y = -4x^5 + 108x^3 - 2x$$

Put in standard form	
Identify leading term	
Identify $\boldsymbol{a}$ *and* $\boldsymbol{n}$	a $n =$
Reference chart and state end behavior.	$as\ x \to \infty, y \to$ $as\ x \to -\infty, y \to$