

Matrices

2.3

9/8/15

A **matrix** is a rectangular array of terms. Matrix elements are arranged in rows and columns. A matrix with m rows and n columns is a **matrix**. The **dimensions** of the matrix are m and n . Matrices can contain any type of numbers or other elements.

2 × 2 matrix

$$\begin{bmatrix} -\frac{3}{5} & \frac{1}{2} \\ 3 & -\frac{3}{4} \end{bmatrix}$$

2 × 5 matrix

$$\begin{bmatrix} 0.2 & 3.4 & -1.5 & 2.1 & 0.8 \\ 3.4 & -3.4 & -2.1 & 1.2 & 0.5 \end{bmatrix}$$

An element of an $m \times n$ matrix can be represented using double subscript notation. a_{24} would be the element in the second row and 4th column.

Two Matrices are equal if and only if they have the same dimensions and are identical, element by element.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

a_{ij} is the element in the i th row and the j th column.

Matrices are usually named using capital letters. $A + B$, exists only if the two matrices have the same dimensions. The element in the i th row and the j th column of $A + B$ is $a_{ij} + b_{ij}$.

Addition of Matrices

The sum of two $m \times n$ matrices is an $m \times n$ matrix in which the elements are the sum of the corresponding elements of the given matrices.

I Try:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix}$$

Given	$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix} =$
Add	$\begin{bmatrix} 1 + (-3) & 2 + 4 \\ 3 + (5) & 4 + (-2) \end{bmatrix}$
Simplify	$\begin{bmatrix} -2 & 6 \\ 8 & 2 \end{bmatrix}$

We Try:

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -3 & 5 \end{bmatrix} + \begin{bmatrix} 0 & 0 & -5 \\ -1 & -2 & -3 \end{bmatrix}$$

Given	$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -3 & 5 \end{bmatrix} + \begin{bmatrix} 0 & 0 & -5 \\ -1 & -2 & -3 \end{bmatrix} =$
Add	
Simplify	

Subtraction

$$A - B = A + (-B)$$

I Try:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix}$$

Given	$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix} =$
Distribute the -	$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ -5 & 2 \end{bmatrix} =$
Add	$\begin{bmatrix} 1 + (3) & 2 + (-4) \\ 3 + (-5) & 4 + 2 \end{bmatrix}$
Simplify	$\begin{bmatrix} 4 & -2 \\ -2 & 6 \end{bmatrix}$

We Try:

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -3 & 5 \end{bmatrix} - \begin{bmatrix} 0 & 0 & -5 \\ -1 & -2 & -3 \end{bmatrix}$$

Given	$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -3 & 5 \end{bmatrix} - \begin{bmatrix} 0 & 0 & -5 \\ -1 & -2 & -3 \end{bmatrix}$
Distribute the -	
Add	
Simplify	

I Try:
Solve for x and y

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 + y \\ 3x \end{bmatrix}$$

Given	$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 + y \\ 3x \end{bmatrix}$
Create the linear system	$x = -2 + y$ $y = 3x$
Solve the system	$y = 3(-2 + y)$ $y = -6 + 3y$ $-2y = -6$ $y = 3$ $3 = 3x$ $1 = x$ $x = 1$

	$y = 3$
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We Try:
Solve for x and y

$$\begin{bmatrix} 4x \\ 5 \end{bmatrix} = \begin{bmatrix} 15 + x \\ 2y \end{bmatrix}$$

Given	$\begin{bmatrix} 4x \\ 5 \end{bmatrix} = \begin{bmatrix} 15 + x \\ 2y \end{bmatrix}$
Create the linear system	
Solve the system	

We Try:

$$\begin{bmatrix} x + y & 3 \\ y & 6 \end{bmatrix} = \begin{bmatrix} 0 & 2y - x \\ y^2 & 4 - 2x \end{bmatrix}$$

Scalar Product

The product of a scalar k and an $m \times n$ matrix A is denoted by kA . Each element of kA equals k times the corresponding element of A .

I try:

$$A = \begin{bmatrix} -1 \\ 2x \end{bmatrix}$$

Solve for $4A$.

Given	$4 \begin{bmatrix} -1 \\ 2x \end{bmatrix}$
Distribute	$4 \begin{bmatrix} -1 \\ 2x \end{bmatrix} = \begin{bmatrix} 4(-1) \\ 4(2x) \end{bmatrix}$

Simplify	$\begin{bmatrix} -4 \\ 8x \end{bmatrix}$
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We Try:

$$A = \begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix}$$

Solve for $-2A$

Given	$-2 \begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix}$
Distribute	
Simplify	

Multiplication

You can also multiply a matrix by a matrix. For matrices A and B , AB if the number of columns in A is the same as the number of rows in B .

$$\begin{bmatrix} 3 & -8 & 1 \\ 1 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 0 & 2 & 0 & 1 \\ -4 & 0 & -2 & 1 \\ 1 & -3 & -1 & 6 \end{bmatrix} \quad \begin{bmatrix} 5 & 3 & 1 & 0 \\ 6 & 0 & 2 & -3 \\ -5 & 3 & 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

2×3 3×4 3×4 3

Since $3 = 3$, multiplication is possible. Since $4 \neq 3$, multiplication is not possible.

Given

$$A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \text{ and } X = \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \end{bmatrix}$$

$$A \cdot X =$$

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \cdot \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \end{bmatrix} = \begin{bmatrix} a_1x_1 + b_1x_2 & a_1y_1 + b_1y_2 \\ a_2x_1 + b_2x_2 & a_2y_1 + b_2y_2 \end{bmatrix}$$

I Try:

$$\text{Given } A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

Solve for AB

Given	$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$
Setup	$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$
Multiply	

We Try:

$$\text{Given } A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

Solve for BB

Given	$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$
Setup	

Multiply	

You try on your whiteboards

$$\text{Given } A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

Solve for AA

Given	$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$
Setup	
Multiply	

You Try on your whiteboards:

$$\text{Given } A = \begin{bmatrix} -1 & 0 \\ 4 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix}$$

Solve for AB

Given	$A = \begin{bmatrix} -1 & 0 \\ 4 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix}$
Setup	
Multiply	

I Try:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad B = \begin{bmatrix} -2 & 1 \\ 3 & -5 \\ 0 & 4 \end{bmatrix}$$

Solve for AB

Given	$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad B$ $= \begin{bmatrix} -2 & 1 \\ 3 & -5 \\ 0 & 4 \end{bmatrix}$
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Setup	$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 \\ 3 & -5 \\ 0 & 4 \end{bmatrix}$
Multiply	

Exit Slip

Solve with your partner. One person explain. The other person Write. Then switch roles

$$\text{Given } A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

1) Solve for $A+B$

2) Solve for AB

