Independent and Dependent Events
Lesson 31

Probability a ratio where we compare how many times an outcome can occur compared to all possible outcomes.

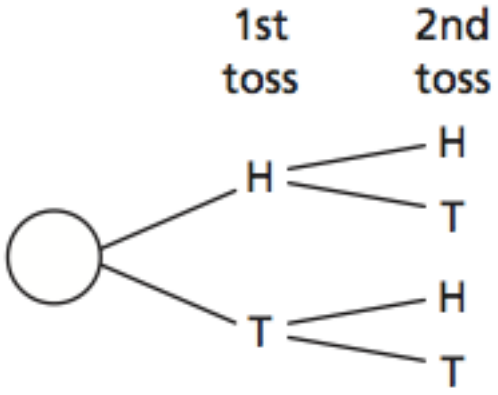
$$\frac{DESIRED}{ALL\ POSSIBILITIES}$$

Events are ***Independent Events*** if the occurrence of one event does not affect the probability of the other.

Probability of Independent Events

If A and B are independent events, then $P(A \text{ and } B) = P(A) \cdot P(B)$.

Coin tosses are independent events.



Finding the Probability of Independent Events

Find each probability.

A **spinning 4 and then 4 again on the spinner**

Spinning a 4 once does not affect the probability of spinning a 4 again, so the events are independent.

$P(4 \text{ and then } 4) = P(4) \cdot P(4)$

$\frac{3}{8} \cdot \frac{3}{8} = \frac{9}{64}$ *3 of the 8 equal sectors are labeled 4.*

B **spinning red, then green, and then red on the spinner**

The result of any spin does not affect the probability of any other outcome.

$P(\text{red, then green, and then red}) = P(\text{red}) \cdot P(\text{green}) \cdot P(\text{red})$

$= \frac{1}{4} \cdot \frac{3}{8} \cdot \frac{1}{4} = \frac{3}{128}$ *2 of the 8 equal sectors are red; 3 are green.*

We Try:

1) What is the probability of rolling a 6 and then an odd number a die?
2) What is the probability of rolling an odd number and an even number on a die.
3) What is the probability of drawing a numbered card in a normal deck of cards and rolling an even number on a die?

You Try:

1) What is the probability of drawing a diamond and flipping heads on a quarter?
2) What is the probability of flipping a tails and then rolling a 7 on a normal 6 sided die?
3) What is the probability of rolling an odd number and drawing a 3 from a deck of 52 cards.

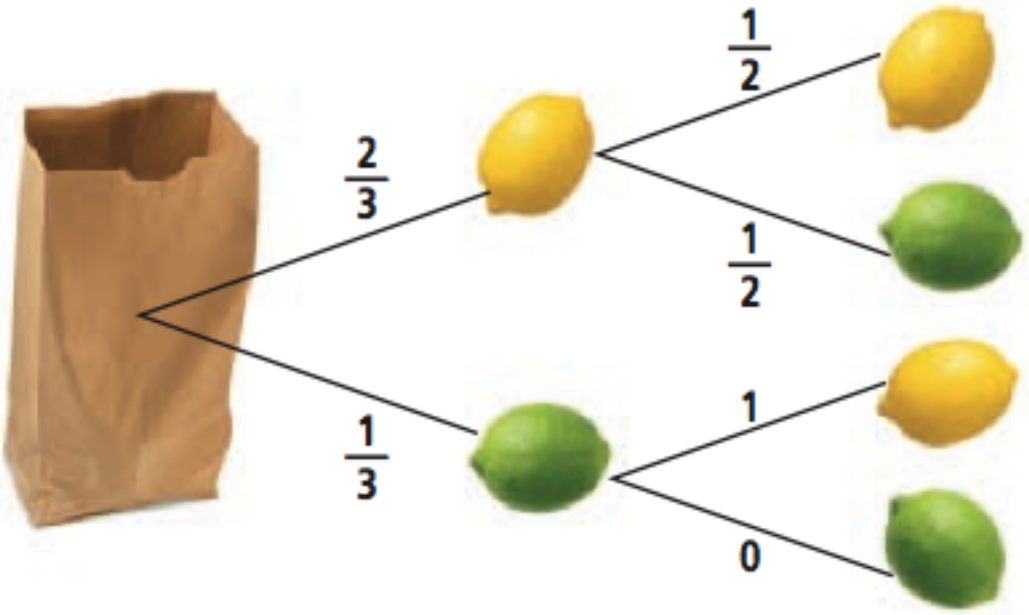
Events are ***Dependent events*** if the occurrence of one event affects the probability of the other.

To find the probability of dependent events, you can use ***Conditional probability*** P(B|A), the probability of event B, given that event A has occurred.

Probability of Dependent Events

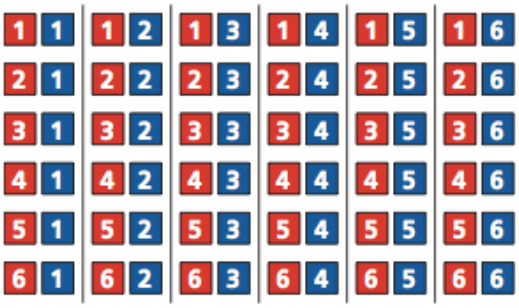
If A and B are dependent events, then $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$, where $P(B \mid A)$ is the probability of B, given that A has occurred.

Choosing 2 pieces of fruit from a bag containing 2 lemons and a lime is a dependent event.


1/2
2/3
1/2
1/3
1
0

Finding the Probability of Dependent Events

Two number cubes are rolled—one red and one blue. Explain why the events are dependent. Then find the indicated probability.



A **The red cube shows a 1, and the sum is less than 4.**

Step 1 Explain why the events are dependent.

$P(\text{red } 1) = \frac{6}{36} = \frac{1}{6}$ *Of 36 outcomes, 6 have a red 1.*

$P(\text{sum} < 4 \mid \text{red } 1) = \frac{2}{6} = \frac{1}{3}$ *Of 6 outcomes with a red 1, 2 have a sum less than 4.*

The events "the red cube shows a 1" and "the sum is less than 4" are dependent because $P(\text{sum} < 4)$ is different when it is known that a red 1 has occurred.

Step 2 Find the probability.

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$

$$P(\text{red } 1 \text{ and sum} < 4) = P(\text{red } 1) \cdot P(\text{sum} < 4 \mid \text{red } 1)$$

$$= \frac{1}{6} \cdot \frac{2}{3} = \frac{1}{18}$$

Explain why the events are dependent. Then find the indicated probability.

B **The blue cube shows a multiple of 3, and the sum is 8.**

The events are dependent because $P(\text{sum is } 8)$ is different when the blue cube shows a multiple of 3.

$P(\text{blue multiple of } 3) = \frac{2}{6} = \frac{1}{3}$ *Of 6 outcomes for blue, 2 have a multiple of 3.*

$P(\text{sum is } 8 \mid \text{blue multiple of } 3) = \frac{2}{12} = \frac{1}{6}$ *Of 12 outcomes that have a blue multiple of 3, 2 have a sum 8.*

$P(\text{blue multiple of } 3 \text{ and sum is } 8) =$

$P(\text{blue multiple of } 3) \cdot P(\text{sum is } 8 \mid \text{blue multiple of } 3) = \left(\frac{1}{3}\right)\left(\frac{1}{6}\right) = \frac{1}{18}$

We try:

1) What is the probability that the red cube shows a number greater than 4, and the sum is greater than 9?

2) What is the probability that the red cube shows a number less than 3, and the sum is less than 5.

3) What is the probability that the red cube is even, and the sum is greater than 6?

You Try:

After two dice, what is the probability that:

1) The first die is 3, and the sum is greater than 5.
2) The first die is 2, and the sum is less than 6.
3) The first die is odd and the sum is 5.

Using a Table to Find Conditional Probability

Largest Texas Counties' Votes for President 2004 (thousands)			
County	Bush	Kerry	Other
Harris	581	472	5
Dallas	345	336	4
Tarrant	349	207	3
Bexar	260	210	3
Travis	148	197	5

The table shows the approximate distribution of votes in Texas' five largest counties in the 2004 presidential election. Find each probability.

A **that a voter from Tarrant County voted for George Bush**

$$P(\text{Bush} \mid \text{Tarrant}) = \frac{349}{559} \approx 0.624$$

Use the Tarrant row. Of 559,000 Tarrant voters, 349,000 voted for Bush.

B **that a voter voted for John Kerry and was from Dallas County**

$$P(\text{Dallas} \mid \text{Kerry}) = \frac{336}{1422}$$

Of 1,422,000 who voted for Kerry, 336,000 were from Dallas County.

$$P(\text{Kerry and Dallas} \mid \text{Kerry}) = \frac{1422}{3125} \cdot \frac{336}{1422} \approx 0.108$$

There were 3,125,000 total voters.

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Harris	581	472	5
Dallas	345	336	4
Tarrant	349	207	3
Bexar	260	210	3
Travis	148	197	5

We Try:

1) Find the probability that a voter from Travis county voted for someone other than George Bush or John Kerry.

2) Find the probability that a voter was from Harris county and voted for George Bush.

Largest Texas Counties' Votes for President 2004 (thousands)			
County	Bush	Kerry	Other
Harris	581	472	5
Dallas	345	336	4
Tarrant	349	207	3
Bexar	260	210	3
Travis	148	197	5

You Try:

1) Find the probability that a person from Bexar voted for Bush.
2) Find the probability that a person who voted for Bush was from Bexar.

In many cases involving random selection, events are independent when there is replacement and dependent when there is not replacement.

Determining Whether Events Are Independent or Dependent

Two cards are drawn from a deck of 52. Determine whether the events are independent or dependent. Find the probability.

A **selecting two aces when the first card is replaced**

Replacing the first card means that the occurrence of the first selection will not affect the probability of the second selection, so the events are independent.

$$P(\text{ace} \mid \text{ace on first draw}) = P(\text{ace}) \cdot P(\text{ace})$$

$$= \frac{4}{52} \cdot \frac{4}{52} = \frac{1}{169}$$ *4 of the 52 cards are aces.*

B **selecting a face card and then a 7 when the first card is not replaced**

Not replacing the first card means that there will be fewer cards to choose from, affecting the probability of the second selection, so the events are dependent.

$$P(\text{face card}) \cdot P(7 \mid \text{first card was a face card})$$

$$= \frac{12}{52} \cdot \frac{4}{51} = \frac{4}{221}$$ *There are 12 face cards, four 7's and 51 cards available for the second selection.*

We Try:
A bag contains 10 beads-2 black, 3 white, and 5 red. A bead is selected at random. Determine whether the events are independent or dependent. Find the probability.

a) Selecting a white bead, replacing it, and then selecting a red bead.
b) Selecting a white bead, not replacing it, and then selecting a red bead.
c) Selecting 3 non-red beads without replacement.

You Try:
A bag contains 10 beads-3 black, 2 white, and 5 red. A bead is selected at random. Determine whether the events are independent or dependent. Find the probability.

a) Selecting a red bead, replacing it, and then selecting a black bead.
b) Selecting a black bead, not replacing it, and selecting a black bead.
c) Selecting 4 non-black beads without replacement.

Closure:

Discuss with your partners.

1) What is the difference between independent and dependent events.
2) You roll two dice. What is probability that the first cube shows a number greater than 4, and the sum is greater than 9?