Ellipses
10.3

An ***Ellipse*** is the set of points in a plane such that the sum of the distances from any point P on the ellipse to two fixed points called the Foci, is constant.

The ***major axis*** is the longer axis.
The ***minor axis*** is the shorter axis.
The ***vertices*** are the endpoints of the major axis.
The ***co-vertices*** are the endpoints of the minor axis.

Section 10.3

Here are some examples of Ellipses.

$$\frac{x^2}{4}+\frac{y^2}{9}=1$$

$$\frac{x^2}{64}+\frac{y^2}{25}=1$$

$$\frac{(x-2)^2}{4}+\frac{(y+1)^2}{9}=1$$

Standard Form for the Equation of an Ellipse Center at (h, k)

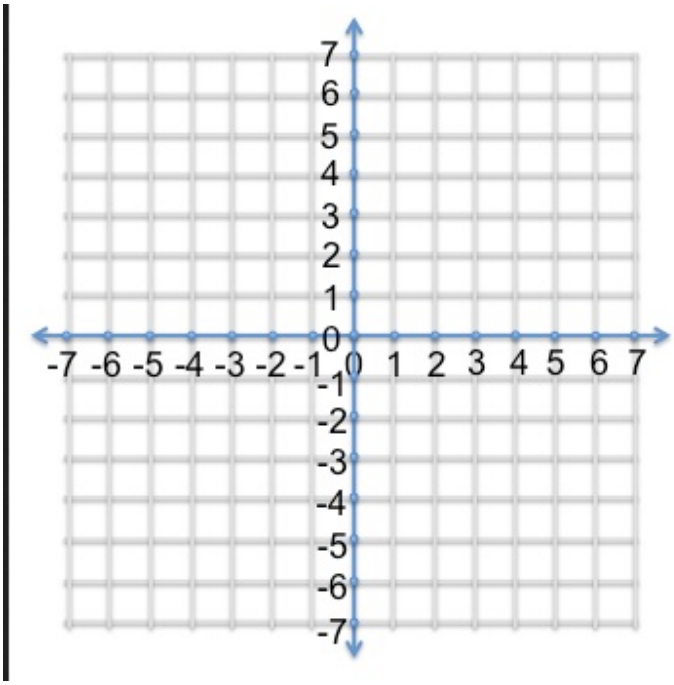
MAJOR AXIS	HORIZONTAL	VERTICAL
Equation	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$	$\frac{(y-k)^2}{a^2}+\frac{(x-h)^2}{b^2}=1$
Vertices	$(h+a, k), (h-a, k)$	$(h, k+a), (h, k-a)$
Foci	$(h+c, k), (h-c, k)$	$(h, k+c), (h, k-c)$
Co-vertices	$(h, k+b), (h, k-b)$	$(h+b, k), (h-b, k)$

Section 10.3

I try

$$\frac{(x)^2}{4}+\frac{(y)^2}{9}=1$$

Identify and plot Center	(0,0)
Identify a, b	$4 = b^2$, $9 = a^2$ $2 = b$, $3 = a$
Identify Major Axis	Vertical
Add and subtract "a" to the center to plot the Vertices. Add and subtract "b" to the center to plot the Co-vertices.	Vertices (0,3) and (0,-3) Co-Vertices (2,0) and (-2,0)
Connect the dots	



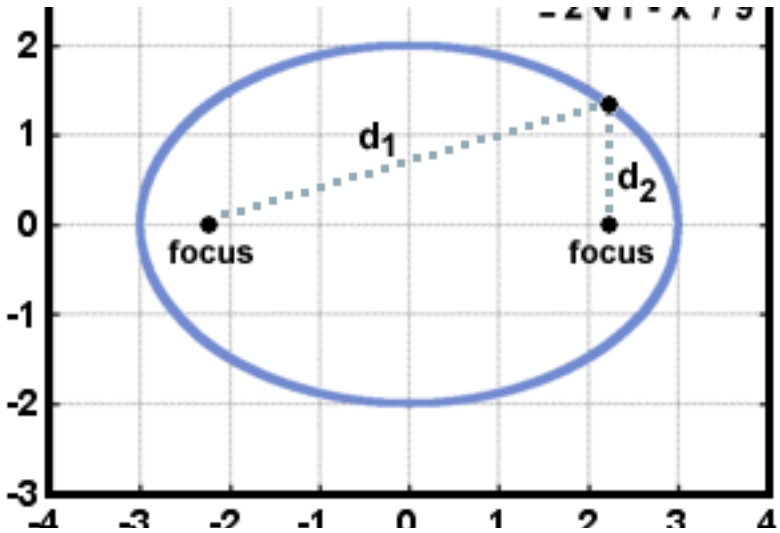
We Try

Section 10.3

$$\frac{(x-2)^2}{1}+\frac{(y-3)^2}{5}=1$$

Identify and plot Center	
Identify a, b	
Identify Major Axis	
Add and subtract "a" to the center to plot the Vertices. Add and subtract "b" to the center to plot the Co-vertices.	Vertices Co-Vertices
Connect the dots	

- The two fixed points are called the **foci** (plural of **focus**) of the ellipse.



- The point halfway between the foci is the **center** of the ellipse.
- The line segment containing the foci of an ellipse with both endpoints on the ellipse is called the **major axis**.

Standard Form Of Ellipses

Horizontal Major Axis		Vertical Major Axis
$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$	***or***	$\frac{(y-k)^2}{a^2}+\frac{(x-h)^2}{b^2}=1$
$\frac{(x-1)^2}{9}+\frac{(y-2)^2}{4}=1$	***or***	$\frac{(y-2)^2}{9}+\frac{(x-1)^2}{4}=1$

Take a minute and discuss the differences and similarities between the horizontal and vertical ellipses with your partner.

Where are the vertices located? How do you know?
What is the relationship between “a” and “b”?
What are “a” and “b” and what do they tell us about the ellipse?

A tells us the distance from the vertex to the center.
B tells us the distance from the co-vertex to the center.
C tells us the distance from the focus to the center.

Identify Major axis.
Co-vertex lies on the…minor
Vertex lies on the… major
Focus lies on the…major

Is the major axis horizontal or vertical given center is at (0,0)

How to tell. Plot the point. Draw the line from the point to the center. Is it vertical or horizontal. Is it on the major or minor.

We will try to get 5 correct in a row.

If the vertex is (5,0).

If the Co-vertex is (5,0)

If the Focus is (5,0)

If the Vertex is (0,1)

If the Co-vertex is (-2,0)

If the Focus is (0,-2)

Graph and write the equation the ellipse with center (0,0), Vertex (10,0), and Co-Vertex (0,8).

We can also find the focus!

$$To\ find\ the\ Focus\ distance, (c), use\ the\ equation: c^2 = a^2 - b^2$$

Find the focus distance for $\frac{x^2}{16}+\frac{y^2}{25}=1$

$$a^2 = 25$$
$$b^2 = 16$$
$$c^2 = 25\text{-}16$$
$$c^2 = 9$$
$$c = 3$$

<u>To find the foci coordinate points, add and subtract the focus value to the center.</u>

$$\frac{x^2}{16}+\frac{y^2}{25}=1$$

Identify Center	(0,0)
Identify Major Axis	Vertical axis $\frac{x^2}{16}+\frac{\mathbf{y^2}}{\mathbf{25}}=1$
Add and subtract focus value to the x or y value of the center to find the Foci. (Whichever one is the major axis)	Foci is at. (0,3) and (0,-3)

<u>I try:</u>

<u>Example #1:</u>

Write an equation in standard form for each ellipse with center (0, 0) and Vertex at (6, 0); focus at (3,0)

Identify givens	**Center: (0,0)** **Vertex: (6,0)** **Focus: (3,0)**

Section 10.3

Identify Major axis	**Horizontal**
Solve to find Co-vertex or vertex using $c^2 = a^2 - b^2$	**Missing: Co-vertex** **C=3** **A=6** $\mathbf{3^2 = 6^2 - b^2}$ $\mathbf{9 = 36 - b^2}$ $\mathbf{-27 = -b^2}$ $\mathbf{27 = b^2}$
Plug parts into standard equation.	$\frac{x^2}{36} + \frac{y^2}{27} = 1$

We Try:

Example #2:

Write an equation in standard form for each ellipse with center (0, 0) and Co-Vertex at (4, 0) and focus at (0, 3).

Identify givens	
Identify Major axis	
Solve to find Co-vertex or Vertex using $c^2 = a^2 - b^2$	
Plug parts into standard equation.	

Write an equation in standard form for each ellipse with center (0, 0) and Co-Vertex at (0, 7) and vertex at (-9,0)).

Identify givens	
Identify Major axis	
Solve to find Co-vertex or Vertex using $c^2 = a^2 - b^2$	
Plug parts into standard equation.	

You Try in your notebooks with your partners:
Write an equation in standard form for each ellipse with center (0,0).

1) Co-vertex (10,0),
focus (0,24)

2) Vertex (-7,0),
focus $(\sqrt{13}, 0)$

Closure
Don't let this happen to you!

What are some common mistakes students might make to solve this problem? Take a minute to solve this problem in your notebook and note any areas people might mess up on!

Write an equation in standard form for each ellipse with center (0, 0) and Co-Vertex at (4, 0) and focus at (0, $3\sqrt{5}$).

Identify givens	
Identify Major axis	
Solve to find Co-vertex or Vertex using $c^2 = a^2 - b^2$	
Plug parts into standard equation.	

Section 10.3
