Fundamental Counting Principle, Permutations, Combinations 11.1

5.16.16

Fundamental Counting Principle

If there are n items and m_1 ways to choose a first item, m_2 ways to choose a second item after the first item has been chosen, and so on, then there are $m_1 \cdot m_2 \cdot \ldots \cdot m_n$ ways to choose n items.

Jason is a co-owner of a restaurant. Help Jason find out the possible lunch special combinations if you can have one of each entrée, side dish, and drink.

#2 I do:

In Utah, a license plate consists of 3 digits followed by 3 letters. The letters *I,O,U* are not used, and each digit or letter may be used more than once. How many different license plates are possible?

#3 We do:

A "Make your own adventure" store lets you choose 6 starting points, gives 4 plot choices, and then has 5 possible endings. How many adventures are there?

#4 You do:

A password is 4 letters followed by 1 digit. Uppercase letters and lowercase letters may be used and are considered different. How many passwords are possible?

n Factorial

For any whole number *n*,

WORDS	NUMBERS	ALGEBRA
The **factorial** of a number is the product of the natural numbers less than or equal to the number. 0! is defined as 1.	$6! =$ $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$	$n! =$ $n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdot \ldots \cdot 1$

$$5! = \frac{6!}{6}$$

I do:

1) $5! \cdot 4! = (5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)(4 \cdot 3 \cdot 2 \cdot 1) = (120)(24) = 2880$

$* \text{ Note: } 5! \cdot 4! \neq 20!$

2) $\frac{5!}{4!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1} = 5$

3) $4! + 2! = (4 \cdot 3 \cdot 2 \cdot 1) + (2 \cdot 1) = 24 + 2 = 26$

4) $6! - 3! = (6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot) - (3 \cdot 2 \cdot 1) = 720 - 6 = 714$

5) $(5 - 3)! \cdot (2 + 1)! = 2! \cdot 3! = 2 \cdot 6 = 12$

We do:

1) $6! \cdot 3! =$

2) $\frac{5!}{2!} =$

3) $2! + 3! =$

4) $5! - 2! =$

5) $\frac{5!}{(5-5)!} =$

You do:

1) $3! + 5! =$

2) $(6-2)! + 3! =$

3) $4! \cdot (5-5)! =$

A ***Permutation*** is a selection of a group of objects in which order is important.

Permutations

NUMBERS	ALGEBRA
The number of permutations of 7 items taken 3 at a time is $_7P_3 = \frac{7!}{(7-3)!} = \frac{7!}{4!}$.	The number of permutations of n items taken r at a time is $_nP_r = \frac{n!}{(n-r)!}$.

I Try:

Solve for $_5P_3$

$$\frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = 5 \cdot 4 \cdot 3 = 60$$

We Try:

Solve for ${}_7P_5$

You Try:

Solve for

1) ${}_8P_6$

2) ${}_8P_8$

3) ${}_8P_1$

In Jason's restaurant, how many people can he choose to for the position of a barista, host, and cook from a group of 6 people?

Does order matter in this case?
Why or why not?

This is selecting and arranging 3 items from 6.

$$_6P_3$$

$$\frac{6!}{(6-3)!} = \frac{6!}{3!} = \frac{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{3\cdot 2\cdot 1} = 6\cdot 5\cdot 4 = 120$$

There are 120 ways to select 3 people.

A ***Combination*** is a grouping of items in which order does not matter.

6 permutations → {ABC, ACB, BAC, BCA, CAB, CBA}

1 combination → {ABC}

To find the number of combinations, the formula for permutations can be modified.

$$\text{number of permutations} = \frac{\text{ways to arrange all items}}{\text{ways to arrange items not selected}}$$

Because order does not matter, divide the number of permutations by the number of ways to arrange the selected items.

$$\text{number of combinations} = \frac{\text{ways to arrange all items}}{(\text{ways to arrange selected items})(\text{ways to arrange items not selected})}$$

Combinations

NUMBERS	ALGEBRA
The number of combinations of 7 items taken 3 at a time is $_7C_3 = \frac{7!}{3!(7-3)!}$.	The number of combinations of n items taken r at a time is $_nC_r = \frac{n!}{r!(n-r)!}$.

Jason is going to adopt fish from a school of 7. How many ways can he choose a group of 4 fish?
Does order matter?

$${}_7C_4$$

$$\frac{7!}{4!\,(7-4)!} = \frac{7!}{4!\,3!} = \frac{7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{(4\cdot3\cdot2\cdot1)(3\cdot2\cdot1)} =$$

$$\frac{7\cdot6\cdot5}{3\cdot2\cdot1} = \frac{210}{6} = 35$$

Closure:
Explain to your partner:

1) What is the difference between permutations and combinations.
2) Create a situation where you have to use combinations to find the possible groupings.
3) Create a situation where you have to use permutations to find the possible groupings.

4) Create a situation where you have to use the Fundamental Counting Principle to find the possible groupings.