Double-Angle and Half-Angle Identities
14-5
Warm-up

Simplify

1) $\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}$

2) $\sqrt{\frac{2+\frac{\sqrt{3}}{2}}{2}}$

3) $\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{1-\frac{\sqrt{2}}{2}}}$

Double-Angle Identities		
$\sin 2\theta = 2\sin\theta\cos\theta$	$\cos 2\theta = \cos^2\theta - \sin^2\theta$ $\cos 2\theta = 2\cos^2\theta - 1$ $\cos 2\theta = 1 - 2\sin^2\theta$	$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

I Do:

Prove:

$$\sin^2\theta = \frac{1}{2}(1 - cos2\theta)$$

$\sin^2\theta = \frac{1}{2}(1 - cos2\theta)$	Given (Change right)
$= \frac{1}{2}(1 - (1 - 2\sin^2\theta)$	Double angle identity
$= \frac{1}{2}(1 - 1 + 2\sin^2\theta)$	Distribute -1
$= \frac{1}{2}(2\sin^2\theta)$ $= \sin^2\theta$	Simplify

I Do:

Prove:

$$sin2\theta = \frac{2tan\theta}{(1+\tan^2\theta)}$$

$sin2\theta = \frac{2tan\theta}{(1+\tan^2\theta)}$	Given (Change right)
$=\frac{2tan\theta}{\sec^2\theta}$	Pythagorean Identity
$= \frac{2\left(\frac{sin\theta}{cos\theta}\right)}{\frac{1}{\cos^2\theta}}$	Inverse and ratio identity
$=2\left(\frac{sin\theta}{cos\theta}\right)\cdot\frac{\cos^2\theta}{1}$ $= 2sin\theta cos\theta$	Multiply
$= sin2\theta$	Double Angle Identity

We Do:

Prove:

$$(cos\theta + sin\theta)^2 = 1 + sin2\theta$$

$(cos\theta + sin\theta)^2 = 1 + sin2\theta$	Given (Change left)

Half-Angle Identities

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} \qquad \cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}} \qquad \tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

Choose + or − depending on the location of $\frac{\theta}{2}$.

I Do:

Use half-angle identity to find the exact value of each trigonometric expression.

$\cos 165°$

$cos165°$	Given
$= \cos\frac{330}{2}°$	
$=-\sqrt{\frac{1+cos330}{2}}$	Half Angle Identity. Cos is negative in Quadrant 2.
$=-\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}}$	Substitute $cos330° = \frac{\sqrt{3}}{2}$
$=-\sqrt{\frac{\frac{2}{2}+\frac{\sqrt{3}}{2}}{2}} =-\sqrt{\frac{2}{4}+\frac{\sqrt{3}}{4}}$	Simplify.
$=-\frac{\sqrt{2+\sqrt{3}}}{4}$	Simplify.

We Do:

$$\tan\left(\frac{3\pi}{8}\right)$$

$\tan\left(\frac{3\pi}{8}\right)$	Given

Closure:
Exit Slip
In Pairs
Use the half-angle identity to find the exact value of each trigonometric expression

$$\sin\left(\frac{\pi}{8}\right)$$