

Sum and Difference Identities

14-4

| Sum and Difference Identities | |
|---|---|
| Sum Identities | Difference Identities |
| $\sin(A + B) = \sin A \cos B + \cos A \sin B$ $\cos(A + B) = \cos A \cos B - \sin A \sin B$ $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ | $\sin(A - B) = \sin A \cos B - \cos A \sin B$ $\cos(A - B) = \cos A \cos B + \sin A \sin B$ $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$ |

I Do:

Find the exact value of the expression.

$$\sin 75^\circ$$

| | |
|--|---|
| $\sin 75^\circ$ | Given |
| $\sin(30^\circ + 45^\circ)$ | Use 30 and 45 because both sine values are known. |
| $\sin(30^\circ + 45^\circ) =$ $\sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ$ | Use sum Identity |
| $\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$ | Replace with actual values. |
| $\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{2} + \sqrt{6}}{4}$ | Simplify |

I do:

Find the exact value of the expression.

$$\cos\left(-\frac{\pi}{12}\right)$$

| | |
|---|--|
| $\cos\left(-\frac{\pi}{12}\right)$ | Given |
| $\cos\left(-\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{6} - \frac{\pi}{4}\right)$ | Use $\frac{\pi}{6}$ and $\frac{\pi}{4}$ because the values are known |
| $\cos\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}\right)$ | Use sum Identity |
| $\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)$ | Replace with actual values. |
| $\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6} + \sqrt{2}}{4}$ | Simplify |

We Try:
Find the exact value of the expression.

$$\tan 105^\circ$$

| | |
|------------------|--|
| $\tan 105^\circ$ | |
| | |
| | |
| | |
| | |

We Try:

| | |
|--------------------------------------|--|
| $\sin\left(-\frac{11\pi}{12}\right)$ | |
| | |
| | |
| | |
| | |

Proving Identities

I do:

Prove the identity: $\cos(x - \pi) = -\cos x$

| | |
|---|----------------------|
| $\cos(x - \pi) = -\cos x$ | Given (change left) |
| $\cos x(\cos \pi) + \sin x(\sin \pi) = -\cos x$ | Difference Identity |
| $\cos x(-1) + \sin x(0) = -\cos x$ | Plug in known values |
| $-\cos x + 0 = -\cos x$ $-\cos x = -\cos x$ | Simplify |

We try:

Prove $\tan(x - 2\pi) = \tan x$

| | |
|---------------------------|--|
| $\tan(x - 2\pi) = \tan x$ | |
| | |
| | |
| | |
| | |

You Try:

Prove: $\cos\left(x + \frac{\pi}{2}\right) = -\sin x$

| | |
|--|--|
| $\cos\left(x + \frac{\pi}{2}\right) = -\sin x$ | |
| | |
| | |
| | |
| | |