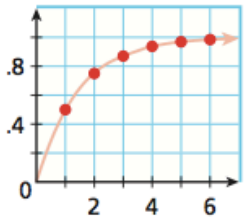
Infinite Geometric Series
12.5

An ***Infinite Geometric Series*** is a geometric series that has infinite terms

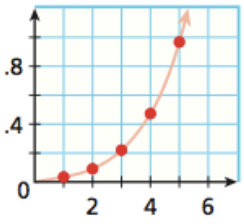
$S_n = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots$

Partial Sums						
n	1	2	3	4	5	6
S_n	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{15}{16}$	$\frac{31}{32}$	$\frac{63}{64}$



$R_n = \frac{1}{32} + \frac{1}{16} + \frac{1}{8} + \frac{1}{4} + \frac{1}{2} + \cdots$

Partial Sums						
n	1	2	3	4	5	6
R_n	$\frac{1}{32}$	$\frac{3}{32}$	$\frac{7}{32}$	$\frac{15}{32}$	$\frac{31}{32}$	$\frac{63}{32}$



What is the common ratio for the first series?

$$r = \frac{a_2}{a_1}$$

What is the common ratio for the second series?

Looking at the two graphs, which would you consider to be "converging" and which would be "diverging"?

A geometric series is considered to be ***Diverging*** if $|r| \geq 1$.

A geometric series is considered to be ***Converging*** if $|r|<1$.

Try:

Determine if the geometric series converges or diverges.

20 + 24 + 28.8 + 34.56+…

Find the common ratio $r = \frac{a_2}{a_1}$	$\frac{24}{20} = \frac{6}{5}$
Diverging if $\|r\| \geq 1$. Converging if $\|r\|<1$.	$\frac{6}{5} > 1$ Diverging

We Try:

Determine if the geometric series converges or diverges.

$1 - 2 + 4 - 8 + 16$+…

Find the common ratio $r = \frac{a_2}{a_1}$	
Diverging if $\|r\| \geq 1$. Converging if $\|r\|<1$.	

Vou Try with your partner.
Even Talk, Odd Write

1) Determine if the geometric series converges or diverges.

$\frac{2}{3} + 1 + \frac{3}{2} + \frac{9}{4} + \frac{27}{8} + \cdots$

Find the common ratio $r = \frac{a_2}{a_1}$	
Diverging if $\|r\| \geq 1$. Converging if $\|r\| < 1$.	

2) Determine if the geometric series converges or diverges.

Even Write, Odd Talk

32 + 16 + 8 + 4 + 2 + …

Find the common ratio $r = \frac{a_2}{a_1}$	
Diverging if $\|r\| \geq 1$. Converging if $\|r\| < 1$.	

f an infinite geometric series converges, we can find the sum of it!

Sum of an Infinite Geometric Series

The sum of an infinite geometric series S with common ratio r and $|r| < 1$ is

$$S = \frac{a_1}{1 - r},$$

where a_1 is the first term.

Try:

Find the sum of the geometric series if it exists.

5 + 4 + 3.2 + 2.56 +...

Find the common ratio	$r = \frac{a_2}{a_1} = \frac{4}{5}$
Does it converge?	$\frac{4}{5} < 1$ Converges
Plug $a_1 and$ "r" into $S = \frac{a_1}{1 - r}$	$S = \frac{5}{(1 - \frac{4}{5})}$
Simplify	$S = (\frac{5}{\frac{1}{5}}) = 25$

*W*e Try:

Find the sum of the geometric series if it exists.

$$\sum_{k=1}^{\infty} \frac{2}{3^{k-1}}$$

Find the common ratio	
Does it converge?	
Plug $a_1 and$ "r" into $S = \frac{a_1}{1-r}$	
Simplify	

You Try:

$$\sum_{k=1}^{\infty} 60\left(\frac{1}{10}\right)^{k}$$

Find the common ratio	
Does it converge?	
Plug $a_1 and$ "r" into $S = \frac{a_1}{1-r}$	
Simplify	

Closure:

Discuss with your partner

How can you tell if a geometric series converges or diverges?

How do we find the common ratio of a geometric series?

What is the formula for the sum of convergent geometric series?

Can arithmetic series converge and diverge?